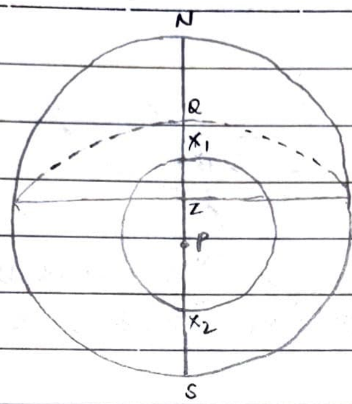
CIRCUMPOLAR BODIES

No. 68 Date 03·04·2025

Q·1) A star when on the meridian below the pole bore south with an altitude 32° 06' and when on the meridian above the pole bore north with altitude 74° 22'. Calculate the observer's latitude & declination of the star.

Soln:- UMP, above the pole = X_1 (bore north)
LMP, below the pole = X_2 (bore south)



$P \rightarrow Z \rightarrow X_1$

pole to observer

Z = zenith, observer

P = pole

QX_1 = declination

Since, PQ = 90°
$\therefore QX_1 = 90 - PX_1$

SP = Latitude N/S
(e)pole ke side waala — same as pole

$NX_1 = 74° 22'$
$SX_2 = 32° 06'$

* $X_1X_2 = 180° - (NX_1 + SX_2)$ (diff name)
$= 180° - (74° 22' + 32° 06')$
$= 180° - 106° 28'$
$= 73° 32'$

PX_1 or $PX_2 = \dfrac{X_1X_2}{2}$

$= \dfrac{73° 32'}{2} = 36° 46'$

Declination, $QX_1 = 90 - PX_1$
$= 90 - 36° 46'$
$= 53° 14'$ (S) → same as lat

Latitude, $SP = SX_2 + PX_2$
$= 32° 06' + 36° 46'$
$= 68° 52'$ (S) → same as pole

No. 69 Date 03.04.25

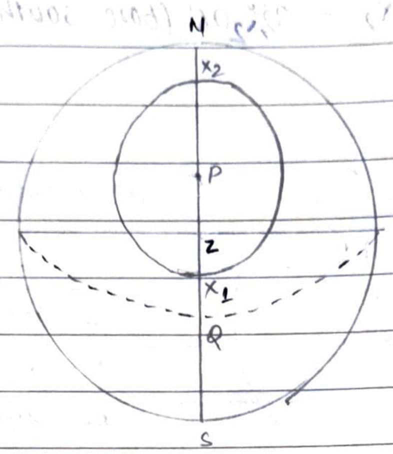
Q.2) An observer obtains the meridian altitude of a body as follows: above the pole 88° 00' to the south and below the pole 10° 15' to the north. Calculate latitude & declination

Soln:-

UMP, above the pole = X_1 (bore south)

LMP, below the pole = X_2 (bore north)

$P \rightarrow Z \rightarrow X_1$



$SX_1 = 88° \, 00' \, S$

$NX_2 = 10° \, 15' \, N$

$X_1X_2 = 180° - (SX_1 + NX_2)$ (diff. name)

$= 180° - (88° + 10° 15')$

$= 180° - 98° 15'$

$= 81° 45'$

PX_1 or $PX_2 = \dfrac{X_1X_2}{2}$

$= \dfrac{81° 45'}{2} = 40° \, 52.5'$

Declination, $QX_1 = 90 - PX_1$

$= 90° - 40° \, 52.5'$

$= 49° \, 07.5'$ (N) $\rightarrow$ same as lat

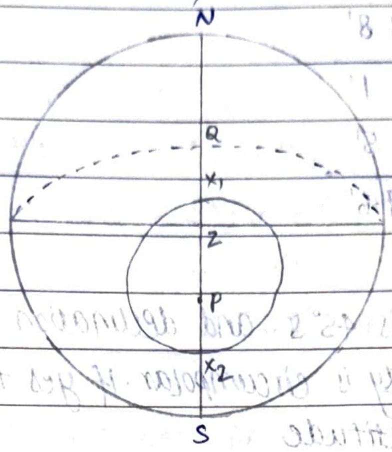
Latitude, $NP = NX_2 + PX_2$

$= 10° 15' + 40° \, 52.5'$

$= 51° \, 07.5'$ (N) $\rightarrow$ same as pole

Q.3) A star with declination 50° 12' S had a true altitude 26° 15' at lower transit. Find the sextant altitude of the same star at upper transit, IE : 1·5' on the arc, H.E : 15·0m

Soln:-



Since, declination is South

I can say, latitude is South. And so, pole in south

$$\text{Declination}, QX_2 = 50°12'$$

$$\text{T.alt@Lower transit}, SX_2 = 26°15'$$

$$\text{Since, Declination}, QX_1 = 90 - PX_1$$

$$50°12' = 90° - PX_1$$

$$PX_1 = 90° - 50°12'$$

$$= 39°48'$$

$$X_1X_2 = PX_1 + PX_2$$

$$= 39°48' + 39°48'$$

$$= 79°36'$$

$$\text{Since, } X_1X_2 = 180° - (SX_2 + NX_1)$$

$$79°36' = 180° - (26°15' + NX_1)$$

$$26°15' + NX_1 = 180° - 79°36'$$

$$26°15' + NX_1 = 100°24'$$

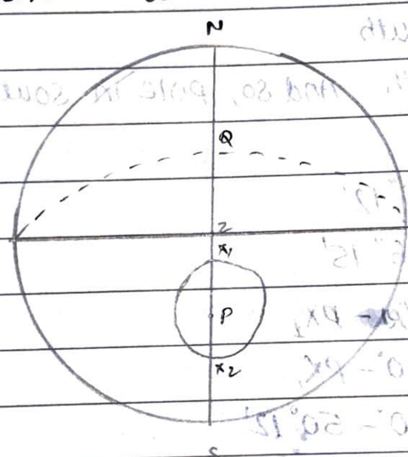
$$NX_1 = 74°09'$$

Hence, True altitude at upper transit = 74° 09'

True altitude : 74° 09'
Total corr^n : (+) 0.3'
App. altitude : 74° 09.3'
Dip (HE - 15.0m) : (+) 6.8'
Obs. altitude : 74° 16.1'
IE (ON) : (+) 1.5'
Sextant altitude : 74° 17.6'

sex. alt
IE ON(-), off (+)
Obs. alt
Dip (H.E.) (-)
App. alt
(HP) Total corr^n (-)
True alt

④ If latitude of observer is 45°S and declination of heavenly body is 50°S. Find if the body is circumpolar. If yes the calculate upper and lower meridian altitude



Declination, $QX_1 = 50°S$; Latitude, $SP = 45°S$

* Condition required for a body to be circumpolar
- Lat + Dec ≥ 90° (For body not to set)
- Lat & dec should be of same name

- So here, Lat + Dec ⇒ 45° + 50°
  = 95° i.e. ≥ 90°

& Lat & dec are both of same name i.e. South
therefore, body is circumpolar

Since, $QX_1 = 90 - PX_1$
$50° = 90° - PX_1$
$PX_1 = 90° - 50°$
$= 40°$

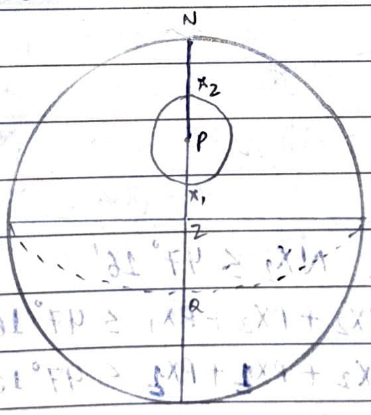
$X_1X_2 = PX_1 + PX_2$
$= 40° + 40°$
$= 80°$

We know, $SP = 45°$
Since, $SP = SX_2 + PX_2$
$45° = SX_2 + 40°$
$SX_2 = 45° - 40°$
$= 05° 00'$ i.e. LMP

Now to find UMP i.e. SX_1
$SX_1 = SX_2 + X_1X_2$
$= 05° 00' + 80° 00'$
$= 85° 00'$ i.e. UMP

⑤ On tropic of cancer, the upper meridian altitude of a star is three times the lower meridian altitude. Find the declination. July 15

Sol^n:- Tropic of cancer = 23° 30'N



Latitude is N, Pole in N

$UMP = 3 \times LMP$
$NX_1 = 3 \times NX_2$
$NX_2 + PX_1 + PX_2 = 3 \times NX_2$
$PX_1 + PX_2 = 2NX_2$
$PX_2 + PX_2 = 2NX_2$ (since $PX_1 = PX_2$)
$2PX_2 = 2NX_2$
$PX_2 = NX_2$

Since, we know NP i.e. 23° 30'N

$$NP = NX_2 + PX_2$$

$$23°30' = PX_2 + PX_2 \quad \text{(From last equation, } NX_2 = PX_2\text{)}$$

$$2PX_2 = 23°30'$$

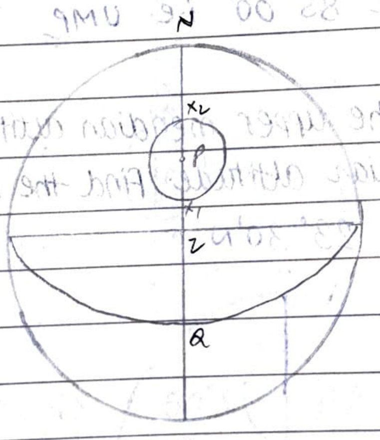
$$PX_1/PX_2 = 11°45'$$

$\therefore$ Declination, $QX_1 = 90° - PX_1$

$$= 90° - 11°45'$$

$$= 78°15' \text{ (N)} \rightarrow \text{same as lat}$$

⑥ For a star to be circumpolar to a observer in a certain North Latitude, Its altitude at upper transit bearing north should not exceed 47° 16'. Find observer's Lat & dec.



From Question, $NX_2 \leq 47°16'$

$$NX_2 + PX_2 + PX_1 \leq 47°16'$$

$$NX_2 + PX_2 + PX_2 \leq 47°16' \quad (PX_1 = PX_2)$$

$$NX_2 + 2PX_2 \leq 47°16'$$

$$NP - PX_2 + 2PX_2 \leq 47°16' \quad (NX_2 = NP - PX_2)$$

$$NP + PX_2 \leq 47°16'$$

$$NP + 90 - dec \leq 47°16' \quad (Dec = 90° - PX_1)$$

$$NP \leq 47°16' - 90° + dec$$

Since, we know body is circumpolar

Hence, Lat + Dec $\geq 90°$

$$47°16' - 90° + dec + dec \geq 90°$$

$47°16' + 2\,dec \geq 90° + 90°$

$47°16' + 2\,dec \geq 180°$

$2\,dec \geq 180° - 47°16'$

$2\,dec \geq 132°44'$

$Dec = 66°22'$ (N) $\rightarrow$ same as Lat

We know, $Lat + dec \geq 90°$

$Lat + 66°22' \geq 90°$

$Latitude \geq 90° - 66°22'$

$Lat = 23°38'\,N$

July'17) A star when on meridian above the pole bore North with a true altitude of 70° 04' and when on meridian below the pole it bore North with a true altitude of 22°05'. Calculate observer's latitude & declination.

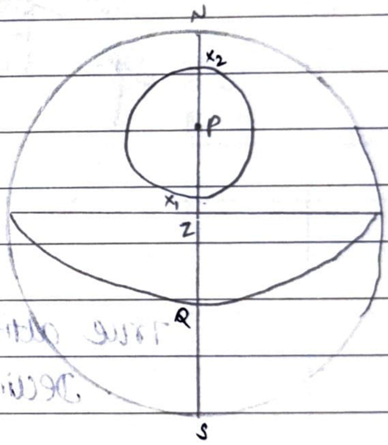
Sol^n:-

UMP, above the pole = X_1 (bore North)

LMP, below the pole = X_2 (bore North)

$NX_1 = 70°04'$

$NX_2 = 22°05'$



from equation, $NX_1 = NX_2 + X_1X_2$

$NX_1 = NX_2 + PX_1 + PX_2$

$NX_1 = NX_2 + 2PX_2 \quad (PX_1 = PX_2)$

$2PX_2 = NX_1 - NX_2$

$2PX_2 = 70°04' - 22°05'$

$2PX_2 = 47°59'$

$PX_2 = 23°59.5'$

$Declination = 90 - PX_2$

$= 90 - 23°59.5'$

$= 66°0.5'\,N$

$Latitude, NP = NX_2 + PX_2$

$= 22°05' + 23°59.5'$

$= 46°04.5'\,N$

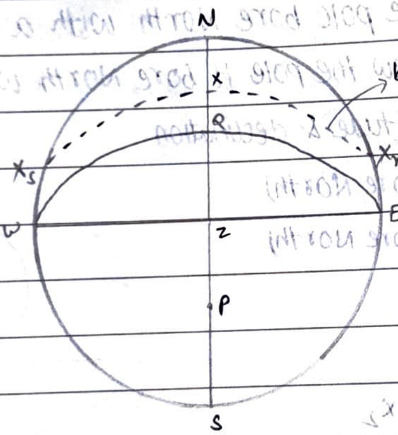
(7) Find two latitudes in which a star having a declination of 68° 46' N will bear north with true altitude of 16° 12'

Sol^n:- Since, declination is 'N', so, when latitude is also in 'N', the body will be circumpolar.

Two Latitude — one in N / one in S

In 'S' Latitude body will not be circumpolar bcz lat & dec become different name

Case (1) : South Latitude

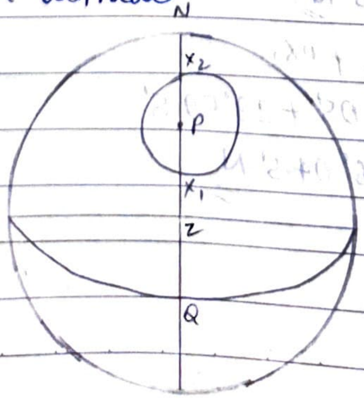


* Latitude = QZ
Declination = QX

True altitude, NX = 16° 12'
Declination, QX = 68° 46' N

$$\text{Latitude}, QZ = 90° - (NX + QX)$$
$$= 90° - (16° 12' + 68° 46')$$
$$= 90° - 84° 58'$$
$$= 05° 02' S$$

Case (2) : North latitude



NP = ?

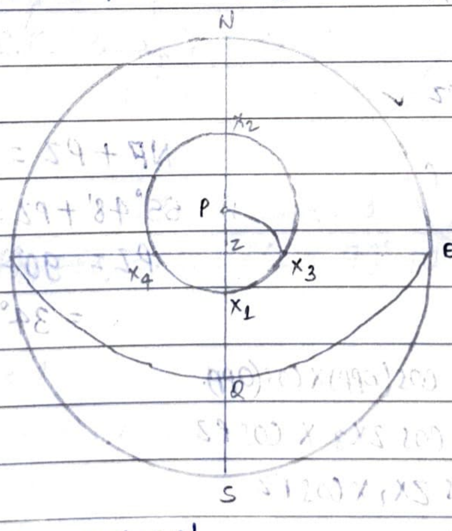
$NX_2 = 16°12'$
$QX_1 = 68°46' N$

Since, Declination, $QX_1 = 90 - PX_1$
$68°46' = 90 - PX_1$
$PX_1 = 90 - 68°46'$
$PX_1 / PX_2 = 21°14'$

Latitude, $NP = NX_2 + PX_2$
$= 16°12' + 21°14'$
$= 37°26' N$

⑧ To a stationary observer the true altitude of a star when on the meridian at the upper transit was 80°09' bearing south and lower transit was 11°45' bearing north. Calculate true altitude when on prime vertical Jan 19

Soln:- UMP, above the pole = X_1 (bore south)
LMP, below the pole = X_2 (bore north)



$SX_1 = 80°09'$
$NX_2 = 11°45'$

$X_1X_2 = 180° - (SX_1 + NX_2)$
$= 180° - (80°09' + 11°45')$
$= 180° - 91°54'$
$= 88°06'$

$PX_1 = PX_2 = PX_3 = \frac{X_1 X_2}{2}$

$= \frac{88^\circ 06'}{2}$

$= 44^\circ 03'$

Declination, $QX_1 = 90^\circ - PX_1$

$= 90^\circ - 44^\circ 03'$

$= 45^\circ 57'$ (N) → same as lat

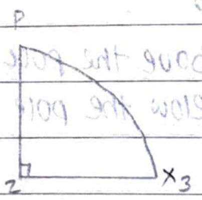
Latitude, $NP = NX_2 + PX_2$

$= 11^\circ 45' + 44^\circ 03'$

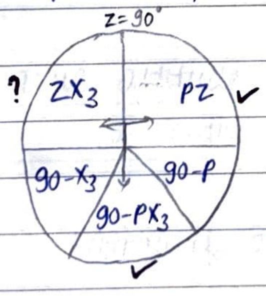
$= 55^\circ 48'$ (N) → same as pole

To find true altitude when on prime vertical

In ΔPZX_3



Using napier's rule



$NP + PZ = 90^\circ$

$55^\circ 48' + PZ = 90^\circ$

$PZ = 90^\circ - 55^\circ 48'$

$= 34^\circ 12'$

$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin(90 - PX_3) = \cos ZX_3 \times \cos PZ$

$\cos PX_3 = \cos ZX_3 \times \cos PZ$

$\cos ZX_3 = \frac{\cos PX_3}{\cos PZ}$

$\cos ZX_3 = \frac{\cos 44^\circ 03'}{\cos 34^\circ 12'}$

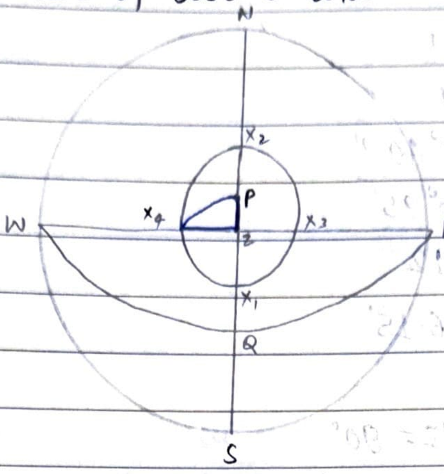
$ZX_3 = 29^\circ 39.4'$

True altitude $= 90^\circ - ZX_3$

$= 90^\circ - 29^\circ 39.4'$

$= 60^\circ 20.6'$

⑨ The sun crosses the observer's prime vertical to the west of the meridian with a true altitude of 14° 25'. If the declination is 10° 19.7'N. Find the latitude of observer and the LHA of the SUN.



From Question, True altitude at prime vertical (west) = 14°25'

That means, True altitude = $90° - ZX_4$

$ZX_4 = 90° - \text{t-alt}$

$ZX_4 = 90° - 14°25'$

$ZX_4 = 75° 35'$

Declination, $QX_1 = 10° 19.7'$ N

Since, we know, $QX_1 = 90 - PX_1$

$10° 19.7' = 90 - PX_1$

$PX_1 = 90° - 10° 19.7'$

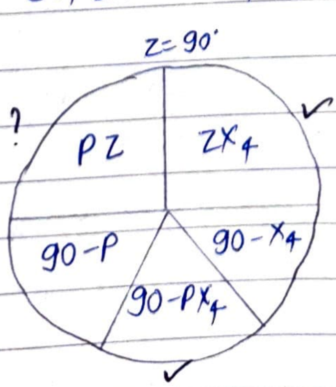
$PX_1 = 79° 40.3'$

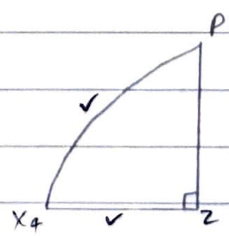
$PX_1 = PX_2 = PX_3 = PX_4 = 79° 40.3'$

To find: NP = ?

For that we require PZ

So, In ΔPZX_4,





$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin(90 - PX_4) = \cos PZ \times \cos ZX_4$

$\cos PX_4 = \cos PZ \times \cos ZX_4$

$\cos PZ = \dfrac{\cos PX_4}{\cos ZX_4}$

$\cos PZ = \dfrac{\cos 79°40.3'}{\cos 75° 35'}$

$\cos PZ = 0.72012$

$PZ = 43° 56.15'$

Since, $NP + PZ = 90°$

$NP + 43° 56.15' = 90°$

$NP = 90° - 43° 56.15'$

Latitude, $NP = 46° 03.85' N$ → same as dec

To find LHA

LHA is the angle between the meridian of your assumed position and the meridian of the geographical position of the celestial body

i.e. $\angle P$

Using Napier's rule

$\sin(\text{mid part}) = \text{Tan}(\text{adj}) \times \text{Tan}(\text{adj})$

$\sin(90 - P) = \text{Tan}\, PZ \times \text{Tan}(90 - PX_4)$

$\cos P = \text{Tan}\, 43° 56.15' \times \text{Tan}(90 - 79° 40.3')$

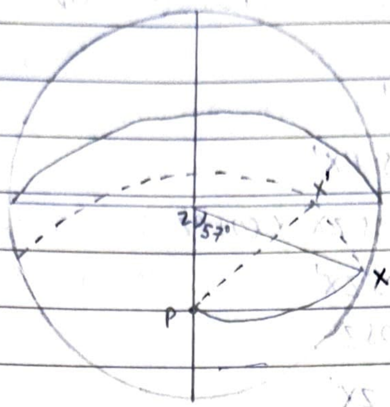
$\cos P = \text{Tan}\, 43° 56.15' \times \text{Tan}\, 10° 19.7'$

$\cos P = 0.1756$

LHA, $P = 79° 53.2'$

(10) APR 2024 An unknown star rose bearing 123°(T) when bearing east it had true altitude of 24° 30'. Find the latitude of observer and star's declination.

Soln.



$ZX' = 90° - \text{true alt.}$

$= 90° - 24°30'$

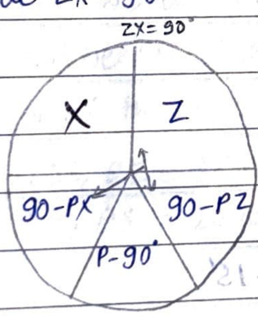
$= 65° \ 30'$

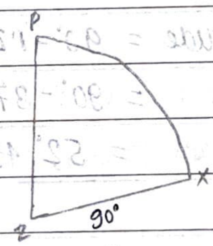
$\angle Z = 123° = S\ 57° E$

In ΔPZX,

Using napier's rule

Side $ZX = 90°$





sin(mid part) = cos (OPP) x cos (OPP)

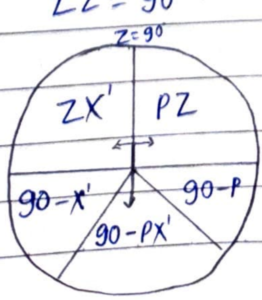
$\sin(90-PX) = \cos Z \times \cos(90-PZ)$

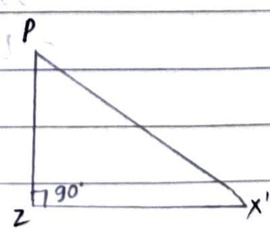
$\cos PX = \cos Z \times \sin PZ$ —— (1)

In $\Delta PZX'$,

Using napier's rule

$\angle Z = 90°$





$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin(90 - PX') = \cos ZX' \times \cos PZ$

$\cos PX' = \cos ZX' \times \cos PZ$ —— (II)

Since, $PX = PX'$

$\cos PX = \cos PX'$

$\cos Z \times \sin PZ = \cos ZX' \times \cos PZ$

$\frac{\sin PZ}{\cos PZ} = \frac{\cos ZX'}{\cos Z}$

$\tan PZ = \frac{\cos ZX'}{\cos Z}$

$\tan PZ = \frac{\cos 65^\circ 30'}{\cos 57^\circ}$

$\tan PZ = 0.7614$

$PZ = 37^\circ 17.15'$

Latitude $= 90^\circ - PZ$

$= 90^\circ - 37^\circ 17.15'$

$= 52^\circ 42.85'$ S

From equation (1),

$\cos PX = \cos Z \times \sin PZ$

$\cos PX = \cos 57^\circ \times \sin 37^\circ 17.15'$

$\cos PX = 0.3299$

$PX = 70^\circ 44.1'$

Declination $= 90^\circ - PX$

$= 90^\circ - 70^\circ 44.1'$

$= 19^\circ 15.9'$ S

(11) Two ship's 'A' and 'B' on the same meridian observe the sun simultaneously at noon and both get the same MZD 40', the declination of SUN is 10°N. The sun bears due north from A and due south from 'B'. Find the latitude of 'A' and 'B'.

Soln:- For vessel 'A' :- brg 000° i.e. Az : 'N'

T. alt : ______ N → Named same as Azimuth

MZD : 40° 00' S → Named opp to T. alt

Dec : 10° 00' N

Latitude : 30° 00' S

For vessel 'B',

brg : 180° i.e. Az : 'S'

T. alt : ______ S

MZD : 40° 00' N } same name add

Dec : 10° 00' N

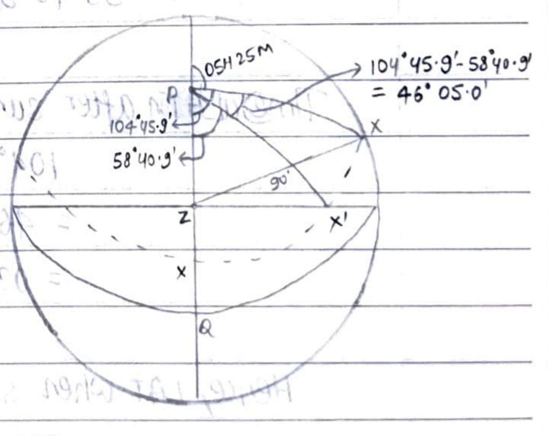
Latitude : 50° 00' N

(12) If the sun rose at 0525 LAT in lat 35°N. Then find the LAT (in A.M), when the sun will be on observer's prime vertical, east of the observer. Given declination of the sun is 20°N.

OCT 2017

Soln:- Given, Lat, QZ = 35°N

Dec, QX = 20°N



Since, PQ = 90°

PZ + QZ = 90°

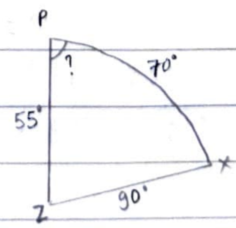
PZ + 35° = 90°

PZ = 55°

Declination = 90° − PX or 90 − PX'

20° = 90° − PX

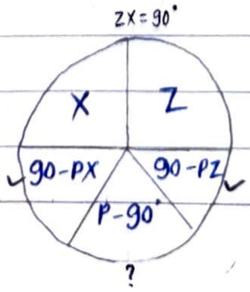
PX' or PX = 70°



In ΔPZX,

side ZX = 90'

Using napier's rule,



No. 83 Date 09.04.

sin (mid part) = Tan(adj) x Tan(adj)

sin (P - 90°) = Tan(90 - PX) x Tan (90 - PZ)

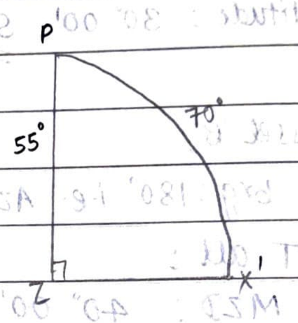
sin (P - 90°) = Tan (90 - 70°) x Tan (90 - 55°)

sin (P - 90°) = Tan 20° x Tan 35°

P - 90 = 14° 45.9'

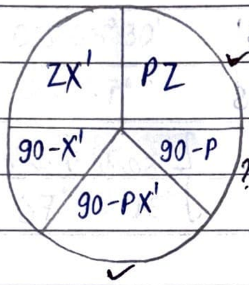
P = 104° 45.9'

When body in on observer's Prime vertical,



In ΔPZX',

Using Napier's rule



sin(mid part) = Tan(adj) x Tan(adj)

sin(90 - P) = Tan (90 - PX') x Tan PZ

cos P = Tan 20° x Tan 55°

P = 58° 40.9'

Duration after sunrise to sun on prime vertical =

104° 45.9' - 58° 40.9'

= 46° 05.0' ÷ 15

= 03h 04m 20s

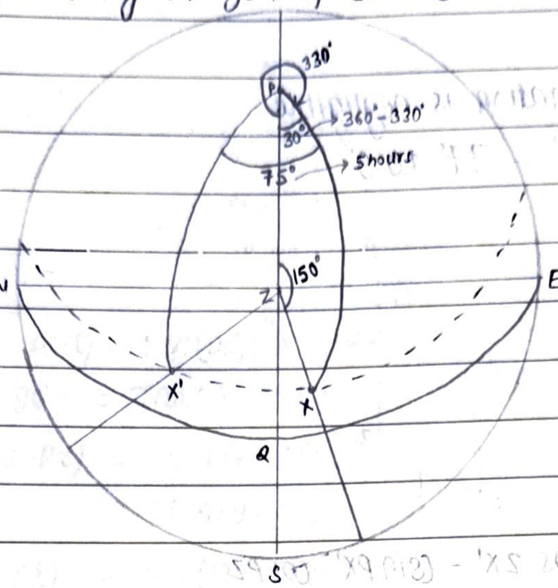
Hence, LAT when sun on prime vertical:

05h 25m 00s + 03h 04m 20s

= 08h 29m 20s

(13) JAN 2010 In Lat 30°00'N, LHA(SUN) 330°, an observer noticed the true altitude and true azimuth of sun 70° and 150° respectively. Find true altitude of SUN 5 hours later, assuming change of sun's Declination is neglible?

Sol:



5 hours = 75°

150+75
= 225°

Given: Lat: 30° 00'N

LHA = 330°

T.alt : 070°

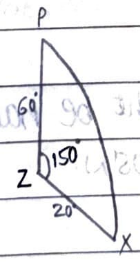
T.AZ = 150°

$PZ = 90° - lat$ $\qquad$ $ZX = 90° - T.alt$

$PZ = 90° - 30°$ $\qquad$ $ZX = 90° - 70°$

$PZ = 60°$ $\qquad$ $ZX = 20°$

In ΔPZX,



$$\cos Z = \frac{\cos PX - \cos PZ \cdot \cos ZX}{\sin PZ \cdot \sin ZX}$$

$\cos Z \cdot \sin PZ \cdot \sin ZX = \cos PX - \cos PZ \cdot \cos ZX$

$\cos PX = \cos Z \cdot \sin PZ \cdot \sin ZX + \cos PZ \cdot \cos ZX$

$\cos PX = \cos 150° \cdot \sin 60° \cdot \sin 20° + \cos 60° \cdot \cos 20°$

$\cos PX = 0.2133$

$PX = 77° 40.9'$

Declination = 90° − PX

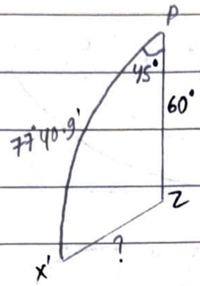
= 90° − 77° 40.9'

= 12° 19.1'

Since declination is negligible

PX = PX' = 77° 40.9'

In ΔPZX',



$$\cos P = \frac{\cos ZX' - \cos PX' \cdot \cos PZ}{\sin PX' \cdot \sin PZ}$$

$\cos P \cdot \sin PX' \cdot \sin PZ = \cos ZX' - \cos PX' \cdot \cos PZ$

$\cos ZX' = \cos P \cdot \sin PX' \cdot \sin PZ + \cos PX' \cdot \cos PZ$

$\cos ZX' = \cos 45° \cdot \sin 77° 40.9' \cdot \sin 60° + \cos 77° 40.9' \cdot \cos 60°$

$\cos ZX' = 0.7049$

TZD, $ZX' = 45° 10.5'$

T. alt $= 90° - ZX'$

$= 90° - 45° 10.5'$

$= 44° 49.5'$

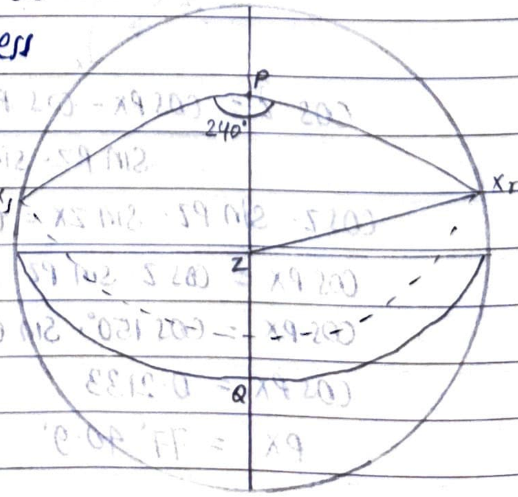
(14) In what latitude will period of night be half of the period of daylight, if declination of sun is 23° 05' N.

JULY 19

Soln:- daylight : darkness

16 hrs : 8 hrs

Duration from sunrise to sunset is 16 hrs i.e. 240°



In ΔPZX,

$\angle P = 240/2$

$= 120°$

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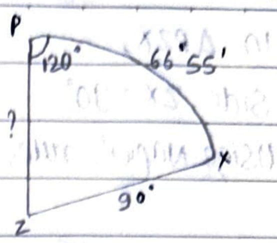
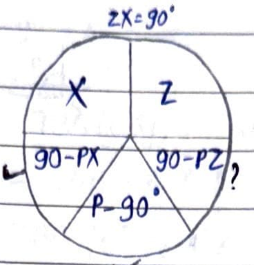
Declination = 23° 05' N

PX = 90° - dec

PX = 66° 55'



Side ZX = 90°



sin(mid part) = Tan(adj) X Tan(adj)

sin (P - 90°) = Tan(90-PX) X Tan (90-PZ)

$$Tan(90-PZ) = \frac{\sin(P-90°)}{Tan(90-PX)}$$

$$Tan(lat) = \frac{\sin(120-90°)}{Tan(90-66°55')}$$

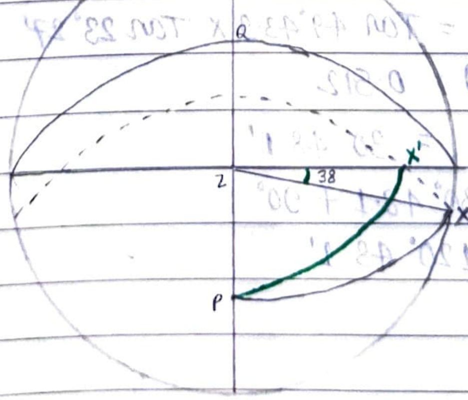
$$Tan(Lat) = \frac{\sin 30°}{\tan 23°05'}$$

Latitude = 49° 33.4' N Ans.

(15) The change in Azimuth of the sun between rising and crossing the prime vertical of a stationary observer is 38°, If the declination of the sun is 23° 27' S. Find observer's latitude and also the change of hour angle of the sun during this period.

JULY 18

Soln.:



PX' or PX = 90 - dec

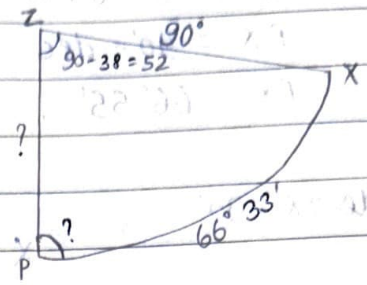
= 90° - 23° 27'

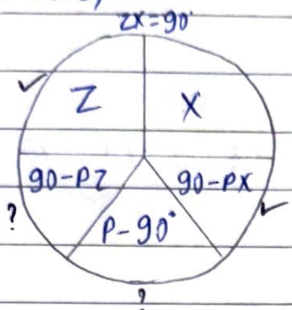
= 66° 33'

In ΔPZX,

Side $ZX = 90^\circ$

Using Napier's rule,





$$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$$

$$\sin(90-PX) = \cos Z \times \cos(90-PZ)$$

$$\cos PX = \cos Z \times \sin PZ$$

$$\sin PZ = \frac{\cos PX}{\cos Z}$$

$$\sin PZ = \frac{\cos 66^\circ 33'}{\cos 52^\circ}$$

$$\sin PZ = 0.646$$

$$PZ = 40^\circ 16.1'$$

$$\text{Latitude} = 90^\circ - PZ$$

$$= 90^\circ - 40^\circ 16.1'$$

$$\boxed{= 49^\circ 43.9' S} \text{ Ans}$$

$$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$$

$$\sin(P-90^\circ) = \tan(90-PZ) \times \tan(90-PX)$$

$$\sin(P-90^\circ) = \tan(90-40^\circ 16.1') \times \tan(90-66^\circ 33')$$

$$\sin(P-90^\circ) = \tan 49^\circ 43.9' \times \tan 23^\circ 27'$$

$$\sin(P-90^\circ) = 0.512$$

$$P - 90^\circ = 30^\circ 48.1'$$

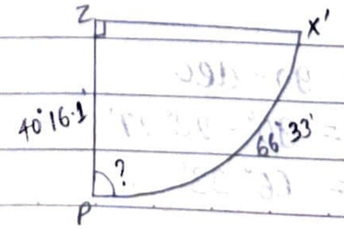
$$P = 30^\circ 48.1 + 90^\circ$$

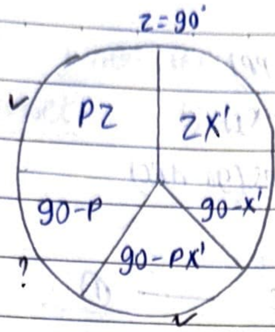
$$P = 120^\circ 48.1'$$

In $\Delta PZX'$

$\angle Z = 90^\circ$

using napier's rule,





$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj}')$

$\sin(90-P) = \tan PZ \times \tan(90-PX')$

$\cos P = \tan 40°\ 16.1' \times \tan 23°27'$

$\cos P = 0.367$

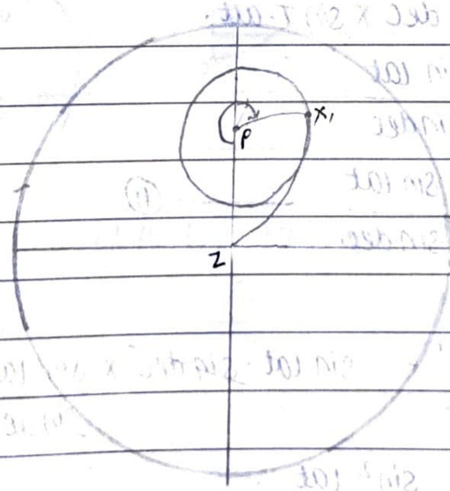
$P = 68°\ 26.5'$

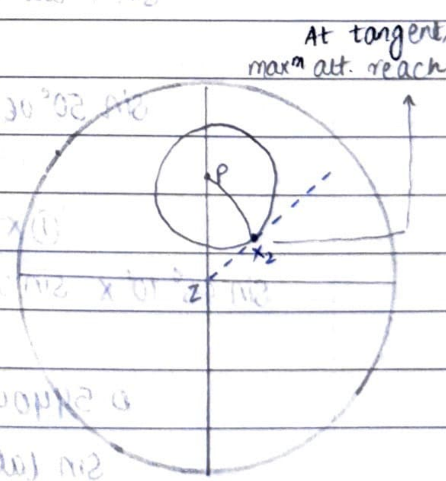
Change in H·A = $120°\ 48.1' - 68°\ 26.5'$

$\boxed{= 52°21.6'}$ Ans

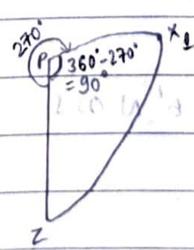
(16) A circumpolar star with LHA 270° has an altitude of 45°10' and later on reaching MAXIMUM AZIMUTH for the same position of observer, it had an altitude of 50°06'. Find the declination of star.

Apr 19



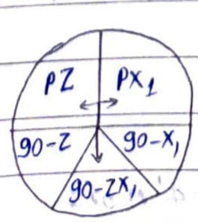


In ΔPZX_1,



$\angle P = 90°$

Using napier's rule,



co lat, $PZ = 90 - lat$ / $Lat = 90 - PZ$
co dec, $PX = 90 - dec$ / $Dec = 90 - PX$
co alt, $ZX = 90 - T.alt$ / $T.alt = 90 - ZX$

$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin(90 - ZX_1) = \cos(PZ) \times \cos(PX_1)$

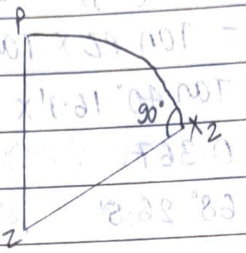
$\sin(90 - ZX_1) = \cos(90 - lat) \times \cos(90 - dec)$

$\sin T.alt = \sin lat \cdot \sin dec$

$\sin 45^\circ 10' = \sin lat \cdot \sin dec$ —— (I)

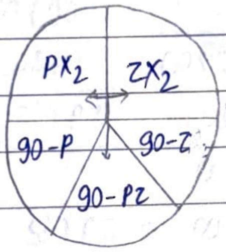
In ΔPZX_2,

$\angle X_2 = 90^\circ$



Note:- The angle b/w tangent & radius is 90° degrees.

Using Napier's rule,



$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos \text{opp}$

$\sin(90 - PZ) = \cos PX_2 \times \cos ZX_2$

$\sin(90 - PZ) = \cos(90 - dec) \times \cos(90 - T.alt)$

$\sin lat = \sin dec \times \sin T.alt.$

$\sin T.alt = \dfrac{\sin lat}{\sin dec}$

$\sin 50^\circ 06' = \dfrac{\sin lat}{\sin dec}$ —— (II)

(I) × (II)

$\sin 45^\circ 10' \times \sin 50^\circ 06' = \sin lat \cdot \cancel{\sin dec} \times \dfrac{\sin lat}{\cancel{\sin dec}}$

$0.54404 = \sin^2 lat$

$\sin lat = 0.7376$

$lat = 47^\circ 31.6'$ N or S

Put lat in Equation (I)

$\sin dec = \dfrac{\sin 45^\circ 10'}{\sin 47^\circ 31.6'}$

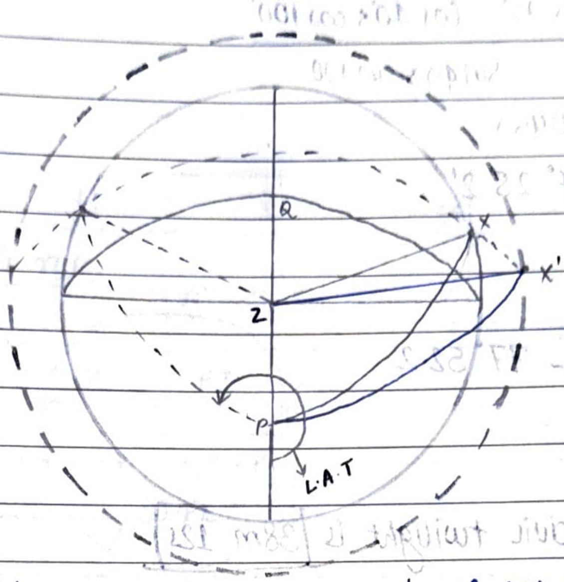
$\sin dec = 0.96145$

Declination $= 74^\circ 02.4'$ N or S

(17) In latitude 50°S, when declination of sun is 10°N. Calculate the following

OCT 2018 JULY 2024

a) L.A.T at sunset b) duration of civil twilight



PZ = 90° − Lat

PZ = 90° − 50°

PZ = 40°

PX = 90° + dec

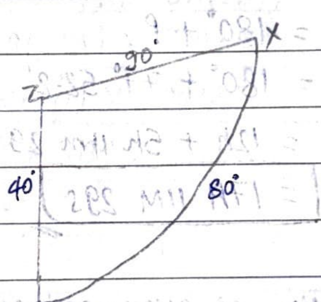
PX = 90° + 10°

PX = 100°

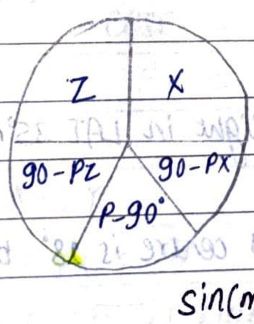
Civil twilight commence when sun is 6° below the horizon

In ΔPZX,

side, ZX = 90°



Using napier's rule,



$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin(P-90°) = \tan(90-PZ) \times \tan(90-PX)$

$\sin(P-90°) = \tan(90-40°) \times \tan(90-100°)$

$\sin(P-90°) = \tan 50° \times \tan -10°$

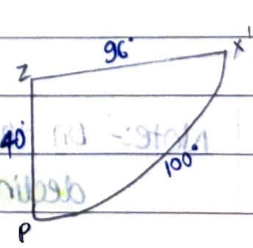
$P-90° = -12° \, 07.8'$

$P = -12° \, 07.8' + 90°$

$P = 77° \, 52.2'$

In ΔPZX'

$ZX' = 90+6 = 96°$



Since, there is no side or angle that is 90°, that is why napier's rule can't apply here.

$$\cos P' = \frac{\cos ZX' - \cos PZ \cdot \cos PX'}{\sin PZ \cdot \sin PX'}$$

$$\cos P' = \frac{\cos 96^\circ - \cos 40^\circ \times \cos 100^\circ}{\sin 40^\circ \times \sin 100^\circ}$$

$\cos P' = 0.0450$

$P' = 87^\circ\, 25.2'$

$P' - P$

$= 87^\circ 25.2' - 77^\circ 52.2'$

$= 9^\circ 33' \div 15$

$= 38m\ 12s$

Duration of civil twilight is $\boxed{38m\ 12s}$

L.A.T at sunset $= 180^\circ + P$

$= 180^\circ + 77^\circ\, 52.2'$

$= 12h + 5h\ 11m\ 29s$

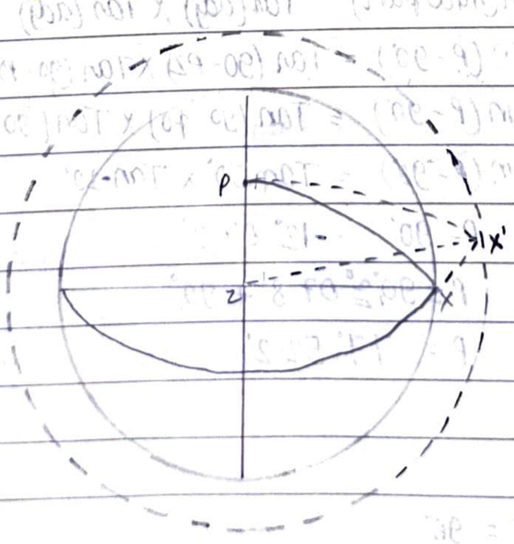
$\boxed{= 17H\ 11M\ 29s}$

(18) Calculate the duration of astronomical twilight in LAT 35°N on the day of spring equinox.

Soln:- Astronomical twilight commences when sun's centre is 18° below the horizon.

Latitude : 35°N

Declination: 0°



Note:- On the day of spring equinox (around March 20th), the sun's declination is 0°

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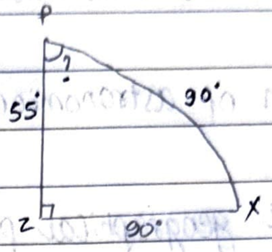
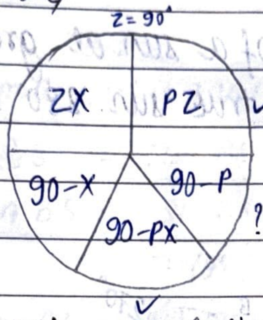
$PZ = 90° - lat$
$= 90° - 35°$
$= 55°$

PX or $PX' = 90° \pm dec$
$= 90° \pm 0$
$= 90°$

In ΔPZX,

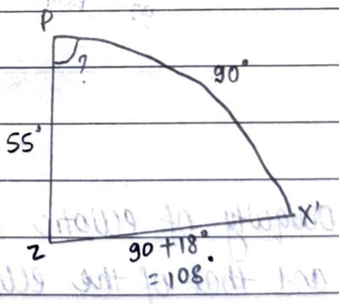


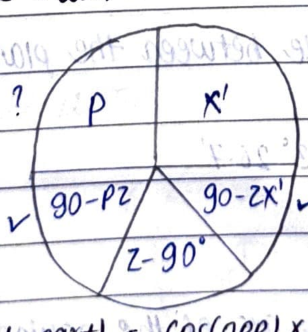
$\angle Z$, PX & $ZX = 90°$
Using napier's rule,



$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$
$\sin(90-P) = \tan(90-PX) \times \tan PZ$
$\cos P = \tan(90-90°) \times \tan 55°$
$\cos P = \tan 0° \times \tan 55°$
$\cos P = 0$
$P = 90°$

In $\Delta PZX'$,
side $PX' = 90°$
Using napier's rule,





$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$
$\sin(90-ZX') = \cos P \times \cos(90-PZ)$
$\cos ZX' = \cos P \times \sin PZ$
$\cos P = \dfrac{\cos ZX'}{\sin PZ}$
$\cos P' = \dfrac{\cos 108°}{\sin 55°}$
$P' = 112° 9.8'$

$P' - P$

$= 112° \; 9.8' - 90'$

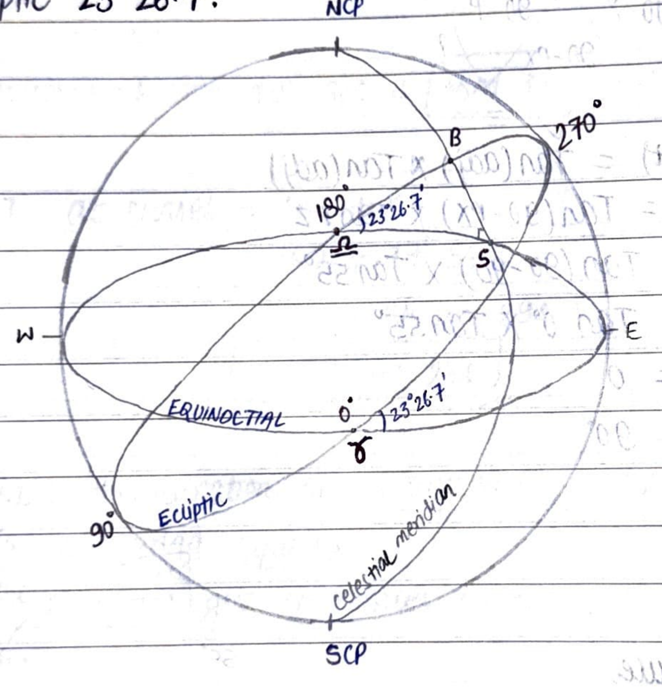
$= 22° \; 9.8' \div 15$

$= 1h \; 28m \; 39s$

Duration of astronomical twilight is 01h 28m 39s Ans.

(19) Calculate geographical position of a sun at greenwich sidereal time 09h 21m 40s, given SHA of true sun 280° 10' and obliquity of ecliptic 23° 26.7'.

OCT 2007



→ Obliquity of ecliptic is the angle between the plane of the Equinoctial and that of the ecliptic

i.e. given in ques = 23° 26.7'

→ ♈ is First point of Aries

→ ♎ is First point of Libra

→ Sidereal hour angle (SHA) is the arc of the equinoctial (or the angle at the celestial pole) contained between the celestial meridian of the First Point of Aries and that through the body, measured westward from Aries.

i.e. given in ques ⇒ ♈S = 280° 10'

♈♎ + S♎ = 280° 10'

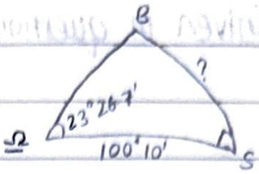
180° + S♎ = 280° 10'

S♎ = 280° 10' − 180°

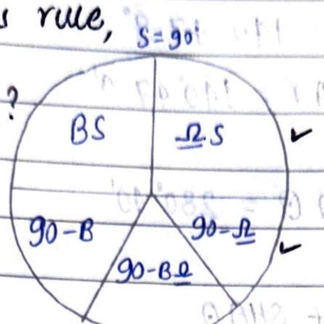
S♎ = 100° 10'

In $\Delta BS\Omega$,

$\angle S = 90°$



Using napier's rule,



$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin \Omega S = \tan BS \times \tan(90-\Omega)$

$\tan BS = \dfrac{\sin \Omega S}{\tan(90-\Omega)}$

$\tan BS = \dfrac{\sin 100°10'}{\tan(90-23°26.7')}$

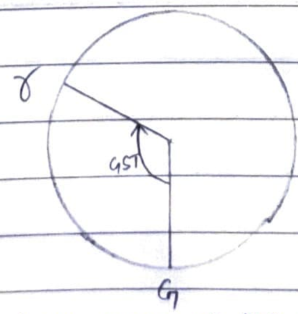
$\tan BS = \dfrac{\sin 100°10'}{\tan 66°33.3'}$

$\tan BS = 0.42686$

Declination, $BS = 23°06.9'N$

→ Declination of a celestial body is the arc of the celestial meridian (or the angle at the centre of the Earth) contained between the Equinoctial and the parallel of declination through that body.

→ Greenwich Sidereal Time



GST is the westerly hour angle of the First point of Aries measured from the greenwich meridian

We know, Sidereal time = 23h 56m 04s (for 360°)

So, 1hr = 15° 02.45'

Given in question,

$GST = 09h\ 21m\ 40s \times 15^\circ\ 02.45'$
$= 140^\circ\ 47.9^\circ$

Hence, $GHA\ \gamma = 140^\circ 47.9'$

Given in question, $SHA\ \odot = 280^\circ\ 10'$

$GHA\ \odot = GHA\ \gamma + SHA\ \odot$
$= 140^\circ 47.9' + 280^\circ 10'$
$= 420^\circ 57.9'$

GHA SUN $= 60^\circ\ 57.9'$ W

GHA 0° to 180° - Long W
GHA 180° to 360° - Long $(360^\circ - GHA)$ E

Note :- (i) Geographical position of the celestial body is defined by equinoctial system of coordinates.
(ii) Since the center of the celestial sphere is the Earth's centre and as the equator and equinoctial are in the same plane, the latitude of a celestial body's GP is equal to body's declination. the longitude of a celestial body's GP corresponds to its GHA

Ans:- Geographical position of SUN :-
Latitude : $23^\circ\ 06.9'$N
Longitude : $060^\circ\ 57.9'$ W