Q.4 Great circle/ Composite great circle

* PRACTISE QUESTIONS

No. 06 Date 26.03.25

Exercise 31

In the following cases, find the initial co, final co, G.C distance and position of vertex along a G.C track

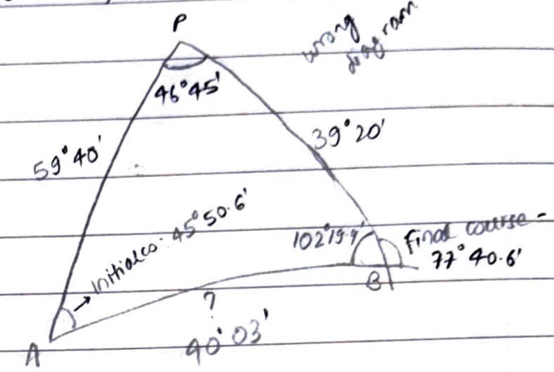
(i) From 30° 20'S 142° 45'W to 50° 40'S 170° 30'E. Also find the lat at which G.C track cuts the meridian of 180°

Sol^n. $\angle P$ = d'long

LONG A = 142° 45' W

LONG B = 170° 30' E

d' long = 46° 45' W



CO-Lat ⇒ PA = 90° − Lat A

= 90° − 30° 20'

= 59° 40'

PB = 90° − Lat B

= 90° − 50° 40'

= 39° 20'

To find GC i.e. AB

$$\cos P = \frac{\cos AB - \cos PA \cdot \cos PB}{\sin PA \cdot \sin PB}$$

$\cos P \cdot \sin PA \cdot \sin PB = \cos AB - \cos PA \cdot \cos PB$

$\cos AB = \cos P \cdot \sin PA \cdot \sin PB + \cos PA \cdot \cos PB$

$\cos AB = \cos 46°45' \cdot \sin 59°40' \cdot \sin 39°20' + \cos 59°40' \cdot \cos 39°20'$

$\cos AB = 0.765$

$AB = 40.05°$

GC dist = 2403.1 NM ($40.05° \times 60$)

To find initial course i.e. $\angle A$

$$\cos A = \frac{\cos PB - \cos AB \cdot \cos PA}{\sin AB \cdot \sin PA}$$

$$\cos A = \frac{\cos 39°20' - \cos 40°3' \cdot \cos 59°40'}{\sin 40°3' \cdot \sin 59°40'}$$

$\cos A = 0.697$; $A = 45° 50.6'$

Initial course = $S\ 45°\ 50.6'\ W$ (If ∠A is less than 90°, Initial co = A & same as pole)

$= 225.8°(T)$

To find final course

we need to find ∠B

$$\cos B = \frac{\cos PA - \cos PB \cdot \cos AB}{\sin PB \cdot \sin AB}$$

$$\cos B = \frac{\cos 59°40' - \cos 39°20' \cdot \cos 40°03'}{\sin 39°20' \cdot \sin 40°03'}$$

$\cos B = -0.213$

$B = 102°19.4'$ ($102.3°$)

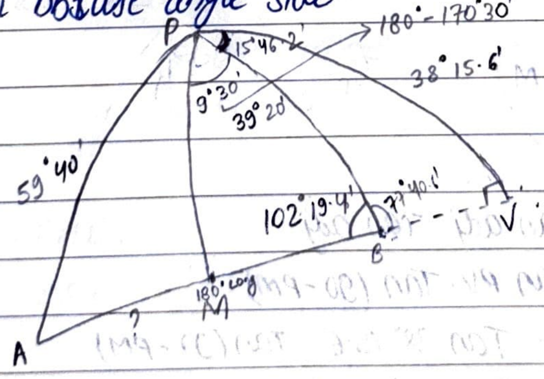
Final co $= S\ 77°\ 40.6'\ W$ (∠B more than 90°, Final co : 180° − B & same as pole)

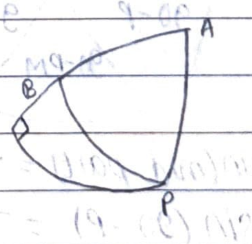
$= 257.7°(T)$

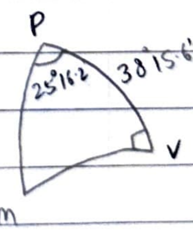
To find vertex

→ Lat A & Lat B are in same hemisphere, so vertex also lie in same hemisphere

→ Since ∠A is acute angle & ∠B is obtuse angle, hence vertex lies on obtuse angle side

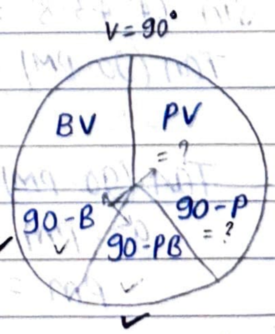






In Δ PBV,

Using napier's rule,



sin(mid part) = cos opp. × cos opp.

$\sin PV = \cos(90 - B) \times \cos(90 - PB)$

$\sin PV = \cos(90 - 102°19.4') \times \cos(90 - 39°20')$

$\sin PV = 0.619$

$PV = 38°15.6'$

Latitude of vertex = $51°\ 44.4'$ S

bcz lat A & lat B are in same hemisphere

sin(mid part) = Tan adj · Tan adj

$\sin(90-P) = \tan(90-PB) \cdot \tan PV$

$\cos P = \tan(90-39°20') \cdot \tan 38°\ 15.6'$

$\cos P = \tan 50°\ 40' \cdot \tan 38°\ 15.6'$

$\cos P = 0.962$

$P = 15°\ 46.2'$

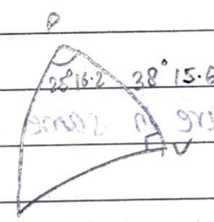
Longitude of vertex = $154°\ 43.8'$ W

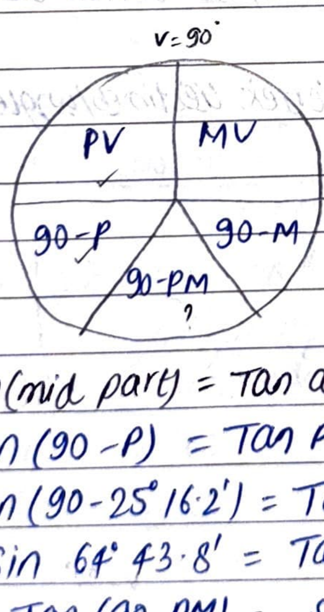
$(170°30' - 15°46.2')$

To find the lat at which G.C track cuts the meridian of 180°

In ΔPMV,

Using napier's rule,





sin(mid part) = Tan adj · Tan adj

$\sin(90-P) = \tan PV \cdot \tan(90-PM)$

$\sin(90-25°\ 16.2') = \tan 38°\ 15.6' \cdot \tan(90-PM)$

$\sin 64°\ 43.8' = \tan 38°\ 15.6' \cdot \tan(90-PM)$

$\tan(90-PM) = \dfrac{\sin 64°\ 43.8'}{\tan 38°\ 15.6'}$

$\tan(90-PM) = 1.147$

$90-PM = 48°\ 54.5'$

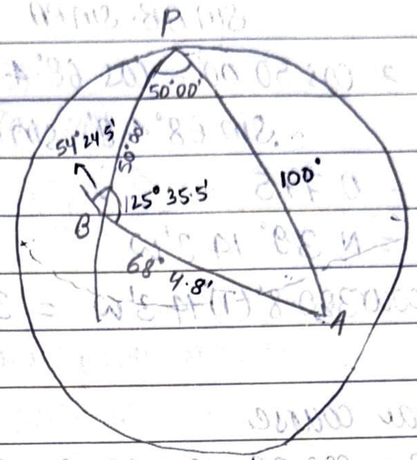
$PM = 41°\ 5.4'$

Latitude $= 90° - 41°\ 5.4'$

$= 48°\ 54.5'$ S

No. 09 Date 26·03·2025

(iii) From 10°00'S, 150°00'W to 40°00'N 160°00'E. Also find the longitude at which G.C track crosses the equator



Long A : 150° 00'W same name −

Long B : 160° 00'E diff name +

d' long : 50° 00'W If substracted by 360°, change the name

co-lat, $PA = 90° + \text{lat } A$

$= 90° + 10°00'$

$= 100°\ 00'$

co-lat, $PB = 90° - \text{lat } B$

$= 90° - 40°00'$

$= 50°\ 00'$

In ΔPAB,

$$\cos P = \frac{\cos AB - \cos PB \cdot \cos PA}{\sin PB \cdot \sin PA}$$

$\cos P \cdot \sin PB \cdot \sin PA = \cos AB - \cos PB \cdot \cos PA$

$\cos AB = \cos P \cdot \sin PB \cdot \sin PA + \cos PB \cdot \cos PA$

$\cos AB = \cos 50°00' \cdot \sin 50°00' \cdot \sin 100°00' + \cos 50°00' \cdot \cos 100°00'$

$\cos AB = 0.373$

$AB = 68°\ 4.8'$

Great circle distance : 4084.8 NM

$(68° 4.8' \times 60)$

To find initial course,

$$\cos A = \frac{\cos PB - \cos AB \cdot \cos PA}{\sin AB \cdot \sin PA}$$

$$\cos A = \frac{\cos 50°00' - \cos 68°4.8' \cdot \cos 100°00'}{\sin 68°4.8' \cdot \sin 100°00'}$$

$\cos A = 0.775$

$A = 39° 14.3'$

Initial course = N39° 14.3' W = 320.8° (T)

(If ∠A less than 90°, Initial co = A & same as pole)

To find Final course,

$$\cos B = \frac{\cos PA - \cos AB \cdot \cos PB}{\sin AB \cdot \sin PB}$$

$$\cos B = \frac{\cos 100°00' - \cos 68°4.8' \cdot \cos 50°00'}{\sin 68°4.8' \cdot \sin 50°00'}$$

$\cos B = -0.582$

$B = 125° 35.5'$

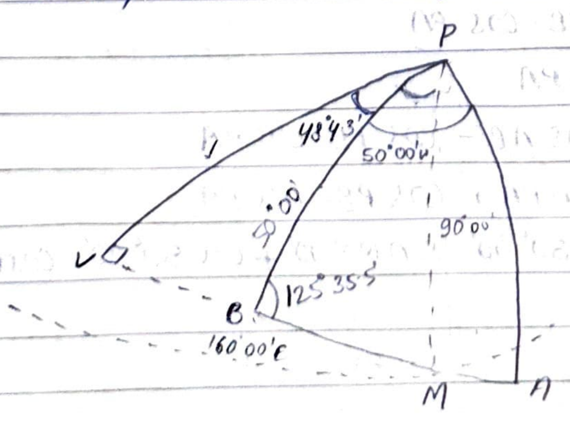
Final course = N 54° 24.5' W (If ∠B is more than 90°, Final co : 180° − B & same as pole)

305.6° (T)

To find position of vertex

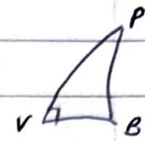
→ Lat A & Lat B are in different hemisphere, but vertex lies in 'NH' bcz north one is higher

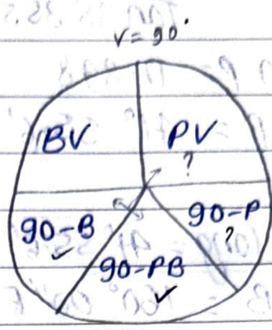
→ Since, ∠B is obtuse angle, vertex will lie on obtuse side.



In Δ PVB,

Using Napier's rule





sin(mid part) = cos opp × cos opp.

$\sin PV = \cos(90-B) \times \cos(90-PB)$

$\sin PV = \cos(90-125°35.5') \times \cos(90-50°00')$

$\sin PV = 0.623$

$PV = 38° 31.9'$

Latitude of vertex = $90° - 38° 31.9'$

$= 51° 28.1'$ N

sin(mid part) = Tan adj × Tan adj

$\sin(90-P) = \tan(90-PB) \times \tan PV$

$\cos P = \tan(90-50°00') \times \tan 38° 31.9'$

$\cos P = \tan 40°00' \times \tan 38° 31.9'$

$\cos P = 0.668$

$P = 48° 4.3'$ W

long B = 160° 00' E

d'long = 48° 4.3' W

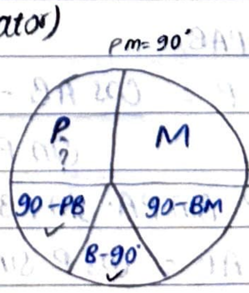
Longitude of vertex = 111° 55.7' E

To find longitude at which a.c track crosses the equator.

In Δ PBM,

Using Napier's rule,

(here side = 90°, bcz at equator)



sin(mid part) = Tan adj × Tan adj

$\sin(90-PB) = \tan P \times \tan(B-90°)$

$$\tan P = \frac{\sin(90-PB)}{\tan(B-90°)}$$

$$\tan P = \frac{\sin(90-50°00')}{\tan(125°35.5'-90°)}$$

$$\tan P = \frac{\sin 40^\circ}{\tan 35^\circ 35.5'}$$

$\tan P = 0.898$

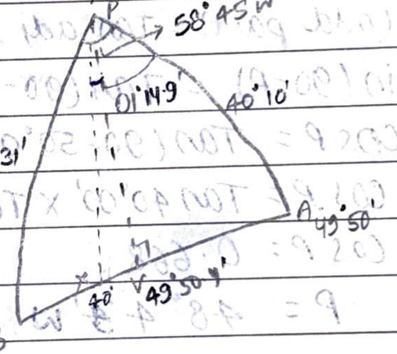
$P = 41^\circ 55.6'$

d' long $= 41^\circ 55.6'$ E

Long B $= 160^\circ 00'$ E

∴ resultant long $= 158^\circ 04.4'$ W

(iv) From $49^\circ 50'$N $005^\circ 15'$W to $32^\circ 29'$N $064^\circ 00'$W. Also find the longitude at which G.C track crosses parallel of 40°N



Posⁿ A : $49^\circ 50'$N $005^\circ 15'$W

$32^\circ 29'$N $064^\circ 00'$W

d'long or LP $= 58^\circ 45'$ W

co-lat PA $= 90^\circ - 49^\circ 50'$

$= 40^\circ 10'$

co-lat PB $= 90^\circ - 32^\circ 29'$

$= 57^\circ 31'$

In ΔPAB,

$$\cos P = \frac{\cos AB - \cos PA \cdot \cos PB}{\sin PA \cdot \sin PB}$$

$\cos P \cdot \sin PA \cdot \sin PB = \cos AB - \cos PA \cdot \cos PB$

$\cos AB = \cos P \cdot \sin PA \cdot \sin PB + \cos PA \cdot \cos PB$

$\cos AB = \cos 58^\circ 45' \cdot \sin 40^\circ 10' \cdot \sin 57^\circ 31' + \cos 40^\circ 10' \cdot \cos 57^\circ 31'$

$\cos AB = 0.6927$

$AB = 46° 9.5'$
$= 2769.5$ NM

To find initial course,

$$\cos A = \frac{\cos PB - \cos PA \cdot \cos AB}{\sin PA \cdot \sin AB}$$

$$\cos A = \frac{\cos 57° 31' - \cos 40° 10' \cdot \cos 46° 9.5'}{\sin 40° 10' \cdot \sin 46° 9.5'}$$

$\cos A = 0.0167$; $A = 89° 02.6'$

Initial Co = N 89° 02.6' W (∠A less than 90°, Initial Co = A & same as pole)

To find final course,

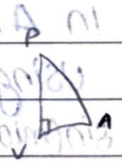
$$\cos B = \frac{\cos PA - \cos PB \cdot \cos AB}{\sin PB \cdot \sin AB}$$

$$\cos B = \frac{\cos 40° 10' - \cos 57° 31' \cdot \cos 46° 9.5'}{\sin 57° 31' \cdot \sin 46° 9.5'}$$

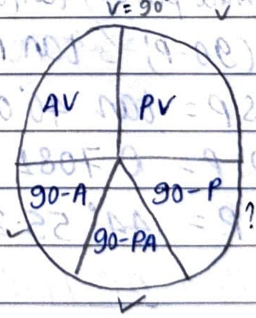
$\cos B = 0.6446$; $B = 49° 51.9'$

Final Co = S 49° 51.9' W (∠B less than 90°, Final Co = B & opp. to pole)

To find position of vertex
⇒ Since ∠A & ∠B both are acute angle, vertex lies in between



In ΔPAV,
Using Napier's rule,



sin (mid part) = tan(adjacent) × tan(adjacent)

$\sin(90-PA) = \tan(90-A) \times \tan(90-P)$

$\cos PA = \tan(90-A) \times \tan(90-P)$

$$\tan(90-P) = \frac{\cos PA}{\tan(90-A)}$$

$$\tan(90-P) = \frac{\cos 40° 10'}{\tan(90-89° 02.8')}$$

$$\tan(90-P) = \frac{\cos 40° 10'}{\tan 0° 57.2'}$$

$\tan(90-P) = 45° 55.4'$

$90-P = 88° 45.1'$

$P = 90° - 88° 45.1'$

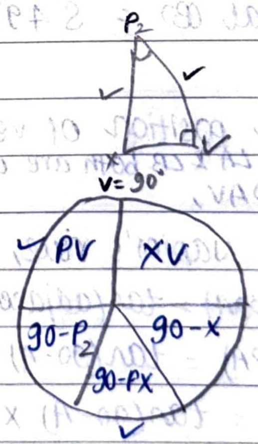
$P_1 = 01° 14.9'$

d'long: 01° 14.9'
Longitude of vertex: 06° 29.9' W

For latitude of vertex,
PV = ?
sin(mid part) = cos opp. × cos opp
$\sin PV = \cos(90-A) \times \cos(90-PA)$
$\sin PV = \cos(90-89°02.8') \times \cos(90-40°10')$
$\sin PV = \cos 0°57.2' \times \cos 49°50'$
$\sin PV = 0.6449$
$PV = 40° 09.6'$

Latitude of vertex = 90 − PV
= 90 − 40° 09.6'
= 49° 50.4' N

To find longitude at which G.C track crosses parallel of 40°N



In ΔPXV,
Using napier rule,
sin(mid part) = tan(adj) × tan(adj)
$\sin(90-P) = \tan PV \times \tan(90-PX)$
$\cos P = \tan 40°09.6' \times \tan 40°$
$\cos P = 0.7081$
$P_2 = 44° 55.2'$
$P_1 = 01° 14.9'$
46° 10.1'

PX = 90° − 40°
= 50°
PV = 90° − 49° 50.4'
= 40° 09.6'

Long A = 005° 15' W
d'long = 46° 10.1'

Resultant long = 51° 25.1' W

No. 15 Date 29·04·25

JAN 2025 A vessel intends to steam a Great circle track from 50°04'N 005°45'W to 47°34'N 052°40'W. Find the G.C distance, initial course, final course & position of the midpoint along the track

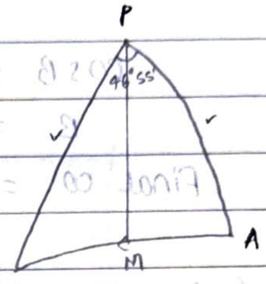
Soln:-

LP = d'long

LONG A : 005°45'W

LONG B : 052°40'W same name (-)

d'long : 046°55'W



CO-lat, PA = 90 − LAT A

= 90 − 50°04'

= 39°56'

CO-lat, PB = 90 − Lat B

= 90 − 47°34'

= 42°26'

In ΔPAB,

$$\cos P = \frac{\cos AB - \cos PB \cdot \cos PA}{\sin PB \sin PA}$$

$\cos P \cdot \sin PB \cdot \sin PA = \cos AB - \cos PB \cdot \cos PA$

$\cos AB = \cos P \cdot \sin PB \cdot \sin PA + \cos PB \cdot \cos PA$

$\cos AB = \cos 46°55' \cdot \sin 42°26' \cdot \sin 39°56' + \cos 42°26' \cdot \cos 39°56'$

$\cos AB = 0.8618$

$AB = 30.48° \; (30°28.9')$

G.C distance = 1828.98 NM

For initial course,

$$\cos A = \frac{\cos PB - \cos PA \cdot \cos AB}{\sin PA \cdot \sin AB}$$

$$\cos A = \frac{\cos 42°26' - \cos 39°56' \cdot \cos 30°28.9'}{\sin 39°56' \cdot \sin 30°28.9'}$$

$\cos A = 0.2372$

$\boxed{A = 76°16.6'}$

Initial CO = N 76°16.6' W

For Final course

$$\cos B = \frac{\cos PA - \cos PB \cdot \cos AB}{\sin PA \cdot \sin AB}$$

$$\cos B = \frac{\cos 39°56' - \cos 42°26' \cdot \cos 30°28.9'}{\sin 42°26' \cdot \sin 30°28.9'}$$

$\cos B = 0.38197$

$B = 67° 32.6'$

Final co = S 67° 32.6' W

To find position of midpoint along the track

In ΔPAM,

$$\cos A = \frac{\cos PM - \cos PA \cdot \cos AM}{\sin PA \cdot \sin AM}$$

$$\cos 76°16.6' = \frac{\cos PM - \cos 39°56' \cdot \cos 15°14.5'}{\sin 39°56' \cdot \sin 15°14.5'}$$

$\cos PM = 0.77985$

$PM = 38°45.2'$

Latitude = $90° - PM$

$= 90° - 38°45.2'$

$= 51° 14.8'$ N

$$\cos P' = \frac{\cos AM - \cos PM \cdot \cos PA}{\sin PM \cdot \sin PA}$$

$$\cos P' = \frac{\cos 15°14.5' - \cos 38°45.2' \cdot \cos 39°56'}{\sin 38°45.2' \cdot \sin 39°56'}$$

$\cos P' = 0.912987$

$P' = 24° 4.7'$

LONG A 005°45' W

dlong 24° 4.7'

LONG M : 029°49.7' W

Position 51° 14.8' N 029° 49.7' W

No. 17 Date 03·06·25

JULY 2024 PM

Find the initial course, final course and GC distance from 75°45'N, 030°46'E to 40°00'N, 110°15'E

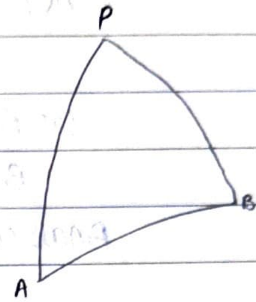
Sol^n:-

$\angle P$ = d'long

LONG A : 030°46'E

LONG B : 110°15'E same name substract

d'long : 79°29'E



CO, Lat PA = 90 - lat A
= 90° - 75°45'
= 14°15'

CO lat, PB = 90° - Lat B
= 90° - 40°00'
= 50°00'

To find G.C distance, AB

$$\cos P = \frac{\cos AB - \cos PA \cdot \cos PB}{\sin PA \cdot \sin PB}$$

$\cos P \cdot \sin PA \cdot \sin PB = \cos AB - \cos PA \cdot \cos PB$

$\cos AB = \cos P \cdot \sin PA \cdot \sin PB + \cos PA \cdot \cos PB$

$\cos AB = \cos 79°29' \cdot \sin 14°15' \cdot \sin 50°00' + \cos 14°15' \cdot \cos 50°00'$

$\cos AB = 0.657$

$AB = 48°53.8'$

$AB = 2933.8$ NM

To find Initial course

$$\cos A = \frac{\cos PB - \cos PA \cdot \cos AB}{\sin PA \cdot \sin AB}$$

$$\cos A = \frac{\cos 50°00' - \cos 14°15' \cdot \cos 48°53.8'}{\sin 14°15' \cdot \sin 48°53.8'}$$

$\cos A = 0.030$

$A = 88°16.3'$

Initial course = N 88°16.3'E (If $\angle A$ is less than 90°, Initial co = A & same as pole)

To find Final course

$$\cos B = \frac{\cos PA - \cos PB \cdot \cos AB}{\sin PB \cdot \sin AB}$$

$$\cos B = \frac{\cos 14°15' - \cos 50°00' \cdot \cos 48°53.8'}{\sin 50°00' \cdot \sin 48°53.8'}$$

$\cos B = 0.947$

$B = 18° 44.1'$

Final co = S 18°44.1'E ($\angle B$ less than 90°; final co = B & opp to pole)

No. 19 Date 03·06·25

JULY 24 AM

A vessel sails along the Great Circle track in Northern Hemisphere changing her longitude by 33°49'. Her initial course was 319°22.8'(T) and her Final course was 298°49.7'(T). Find a) The Great circle distance she sailed
b) The latitudes of departure & arrival positions.

Soln:-

LP = d'long = 33°49'

Initial course = 319° 22.8'

= N 40°37.2'W

same as pole; so ∠A = Initial co

∠A = 40°37.2'

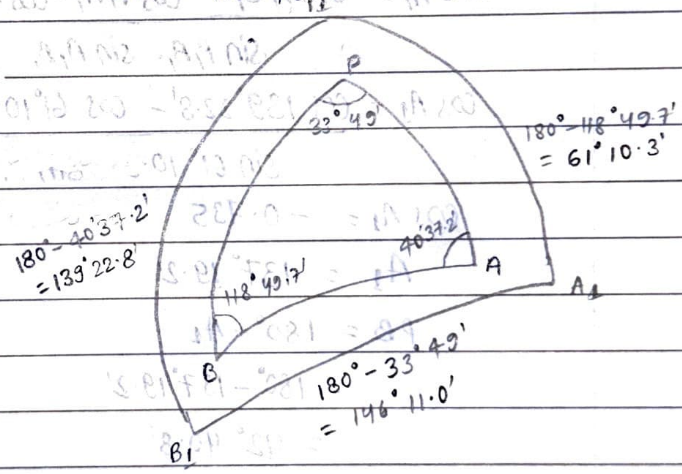
Final course = 298°49.7'

= N 61°10.3'W

same as pole, So Final co = 180° − B

61°10.3' = 180° − B

∠B = 118°49.7'



$$\cos P_1 = \frac{\cos A_1B_1 - \cos P_1A_1 \cos P_1B_1}{\sin P_1A_1 \cdot \sin P_1B_1}$$

$$\cos P_1 = \frac{\cos 146°11.0' - \cos 61°10.3' \cdot \cos 139°22.8'}{\sin 61°10.3' \cdot \sin 139°22.8'}$$

$\cos P_1 = -0.815$

$P_1 = 144° 35.1'$

$AB = 180° - P_1$

$= 180° - 144° 35.1$

$= 35° 24.9'$

AB i.e. GC distance = 2124.9 NM

$$\cos B_1 = \frac{\cos P_1A_1 - \cos P_1B_1 \cdot \cos A_1B_1}{\sin P_1B_1 \cdot \sin A_1B_1}$$

$$\cos B_1 = \frac{\cos 61°10.3' - \cos 139°22.8' \cdot \cos 146°11.0'}{\sin 139°22.8' \cdot \sin 146°11.0'}$$

$\cos B_1 = -0.4097$

$B_1 = 114°11.1'$

$PA = 180° - B_1$

$= 180° - 114°11.1'$

$= 65°48.9'$

Lat $A = 90° - PA$

Lat $A = 90° - 65°48.9'$

$\boxed{\text{Lat } A = 24°11.1'N}$

$$\cos A_1 = \frac{\cos P_1B_1 - \cos P_1A_1 \cdot \cos A_1B_1}{\sin P_1A_1 \cdot \sin A_1B_1}$$

$$\cos A_1 = \frac{\cos 139°22.8' - \cos 61°10.3' \cdot \cos 146°11.0'}{\sin 61°10.3' \cdot \sin 146°11.0'}$$

$\cos A_1 = -0.735$

$A_1 = 137°19.2'$

$PB = 180° - A_1$

$= 180° - 137°19.2'$

$= 42°40.8'$

Lat $B = 90° - PB$

Lat $B = 90° - 42°40.8'$

$\boxed{\text{Lat } B = 47°19.2'}$

Composite great circle

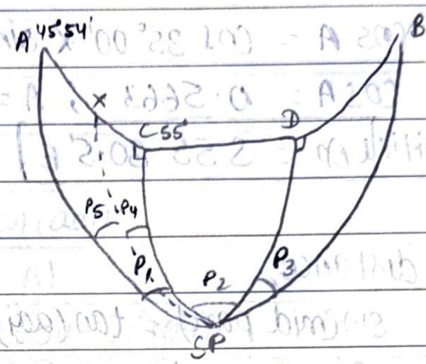
No. 45 Date 29.03.2025

Exercise 58- Composite great circle

① Pos^n A : 45° 54.0'S 170° 45.0'E

Pos^n B : 49° 06.0'S 075° 50.0'W

Max^m Lat: 55°S



co-lat, $PA = 90 - A$
$= 90° - 45° 54.0'$
$= 44° 06.0'$S

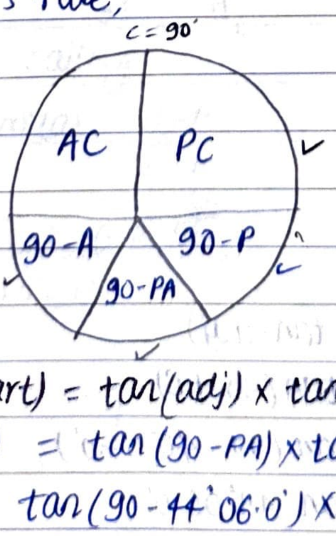
$PB = 90 - B$
$= 90 - 49° 06'$
$= 40° 54.0'$S

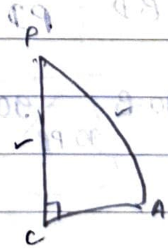
PC or $PD = 90° - 55°$
$= 35° 00'$

LP, d'long = 113° 25'E

In ΔPAC

Using napier's rule,





sin(mid part) = tan(adj) x tan(adj)

$\sin(90 - P) = \tan(90 - PA) \times \tan PC$

$\cos P = \tan(90 - 44° 06.0') \times \tan 35°$

$\cos P = \tan 45° 54.0' \times \tan 35°$

$\cos P = 0.7226$

$P_1 = 43° 44.0'$

For, Initial course, A

sin(mid part) = cos (opp) × cos (opp)

sin(90−A) = cos PC × cos(90−P)

cos A = cos PC × sin P

cos A = cos 35° 00′ × sin 43° 44′

cos A = 0.5663 ; A = 55° 30.5′

$\boxed{\text{Initial Co} = S\ 55°\ 30.5'\ E}$ (If ∠A less than 90°, Initial co = A & same as pole)

For distance,

sin(mid part) = tan (adj) × tan (adj)

sin AC = tan PC × tan (90−A)

sin AC = tan 35° 00′ × tan (90 − 55° 30.5′)

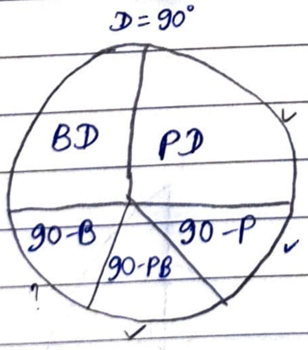
sin AC = tan 35° 00′ × tan 34° 29.5′

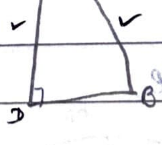
sin AC = 0.4811

AC = 28° 45.4′

AC = 1725.4 NM

In ΔPBD,





Using napier's rule,

sin (mid part) = tan(adj) × tan(adj)

sin(90−P) = tan PD × tan (90−PB)

cos P = tan 35° 00′ × tan (90 − 40° 54.0′)

cos P = tan 35° 00′ × tan 49° 06.0′

cos P = 0.8083

P_3 = 36° 3.9′

sin (mid part) = cos(opp) x cos(opp)

sin(90-B) = cos PD x cos(90-P)

cos B = cos PD x sin P

cos B = cos 35° 00' x sin 36° 3.9'

cos B = 0.4822 ; B = 61° 10.1' (If ∠B less than 90°, final co = B & opp to pole)

$\boxed{\text{Final Co} = N\ 61°\ 10.1'\ E}$

For distance,

sin (mid part) = tan (adj) x tan (adj)

sin BD = tan PD x tan (90-B)

sin BD = tan 35° 00' x tan (90-61° 10.1')

sin BD = tan 35° 00' x tan 28° 49.9'

sin BD = 0.3854

BD = 22° 40.3'

BD = 1360.3 NM

Now for ΔPCD,

$P = P_1 + P_2 + P_3$

113° 25' = 43° 44.0' + P_2 + 36° 3.9'

P_2 = 113° 25' - 43° 44.0' - 36° 3.9'

P_2 = 33° 37.1'

By parallel sailing,

$\cos m'lat = \frac{dep}{d'long}$

Since lat are 55°,

$m'lat = \frac{55+55}{2} = 55°$

d'long = 33° 37.1'

$\cos 55° = \frac{dep}{33° 37.1'}$

dep = cos 55° x 33° 37.1'

dep = 19° 17.0'

= 1156.96 NM

dep = dist

Hence, Distance, CD = 1156.96 NM

Total composite G.C distance

= AC + CD + BD

= 1725.4 + 1156.96 + 1360.3

= 4242.7 NM

Contd.

In above question,

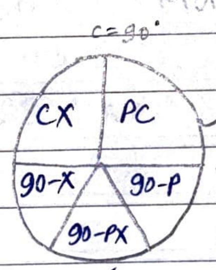
Calculate the longitude in which the track crosses the latitude of 50°S for the first time

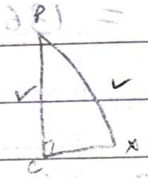
In ΔPCX,

co-lat, $PX = 90° - 50°$

$= 40°00'$

$PC = 35°00'$





sin(mid part) = tan(adj) x tan(adj)

$\sin(90-P) = \tan(90-PX) \times \tan PC$

$\cos P = \tan(90 - 40°00') \times \tan 35°00'$

$\cos P = \tan 50°00' \times \tan 35°00'$

$\cos P = 0.8345$

$P_4 = 33°26.3'$

$P_1 = P_4 + P_5$

$43°44.0' = 33°26.3' + P_5$

$P_5 = 43°44.0' - 33°26.3'$

$= 10°17.7'$

Long A = 170° 45.0' E

d'long = 10° 17.7'

Resultant long = 178° 57.3' W

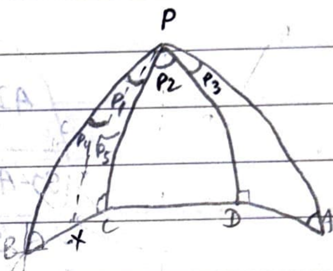
3) Pos^n A : 37° 48.0'N 122° 40.0'W
Pos^n B : 35° 40.0'N 141° 00.0'E

Limiting Lat : 45°N

⇒ d'long = 096° 20.0'W

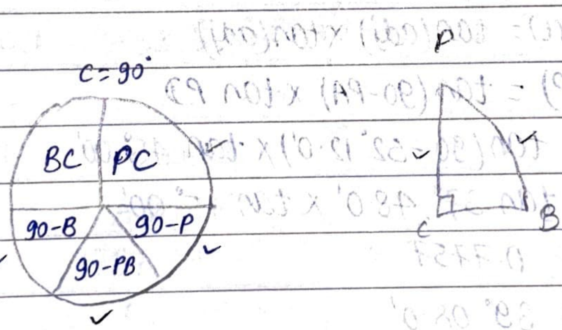
Co-lat, PA = 90 − A
= 90 − 37° 48.0'
= 52° 12.0'

PC & PD = 90 − 45°
= 45° 00'



PB = 90 − B
= 90 − 35° 40'
= 54° 20'

In Δ PBC,



sin(mid part) = Tan(adj) × Tan(adj)
sin(90 − P) = Tan PC × Tan(90 − PB)
cos P = Tan 45° 00' × Tan(90 − 54° 20')
cos P = Tan 45° 00' × Tan 35° 40'
cos P = 0.7177
P_1 = 44° 8.8'

sin(mid part) = cos(opp) × cos(opp)
sin(90 − B) = cos PC × cos(90 − P)
cos B = cos PC × sin P
cos B = cos 45° 00' × sin 44° 8.8'
cos B = 0.4925 ; B = 60° 29.7'

$\boxed{\text{Final co = S 60° 29.7' W}}$ (If ∠B less than 90°, Final co = B & opp to pole)

$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin BC = \tan(90-B) \times \tan PC$

$\sin BC = \tan(90 - 60^\circ 29.7') \times \tan 45^\circ 00'$

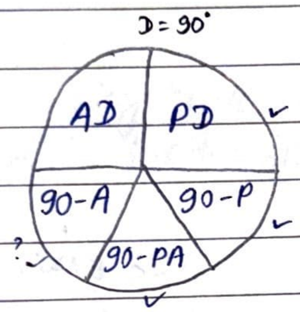
$\sin BC = \tan 29^\circ 30.3' \times \tan 45^\circ 00'$

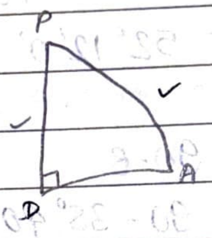
$\sin BC = 0.5659$

$BC = 34^\circ 27.8'$

$= 2067.8$ NM

In ΔPAD,





$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin(90-P) = \tan(90-PA) \times \tan PD$

$\cos P = \tan(90 - 52^\circ 12.0') \times \tan 45^\circ 00'$

$\cos P = \tan 37^\circ 48.0' \times \tan 45^\circ 00'$

$\cos P = 0.7757$

$P_3 = 39^\circ 08.0'$

$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin(90-A) = \cos PD \times \cos(90-P)$

$\cos A = \cos 45^\circ 00' \times \sin 39^\circ 08.0'$

$\cos A = 0.4463$; $A = 63^\circ 29.7'$

$\boxed{\text{Initial Co} = N\ 63^\circ 29.7'\ W}$ If ∠A less than 90°, Initial co = A & same as pole

$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin AD = \tan PD \times \tan(90-A)$

$\sin AD = \tan 45^\circ 00' \times \tan(90^\circ - 63^\circ 29.7')$

$\sin AD = \tan 45^\circ 00' \times \tan 26^\circ 30.3'$

$\sin AD = 0.4997$

$AD = 29^\circ 54.8'$

$= 1794.8$ NM

$$P = P_1 + P_2 + P_3$$

$$096^\circ 20{\cdot}0' = 44^\circ 8{\cdot}8' + P_2 + 39^\circ 08{\cdot}0'$$

$$P_2 = 96^\circ 20{\cdot}0' - 44^\circ 8{\cdot}8' - 39^\circ 08{\cdot}0'$$

$$P_2 = 13^\circ 3{\cdot}2'$$

$$\cos m'lat = \frac{dep}{d'long}$$

$$m'lat = \frac{45+45}{2} = 45^\circ$$

$$d'long = 13^\circ 3{\cdot}2'$$

$$\cos 45^\circ = \frac{dep}{13^\circ 3{\cdot}2'}$$

$$dep = \cos 45^\circ \times 13^\circ 3{\cdot}2'$$

$$dep = 9^\circ 13{\cdot}8'$$

$$CD = 553{\cdot}8 \text{ NM}$$

Total composite G.C distance,

$$AD + CD + BC$$

$$= 1794{\cdot}8 + 553{\cdot}8 + 2067{\cdot}8$$

$$\boxed{= 4416{\cdot}4 \text{ NM}}$$

In above ques., calc^n the latitude in which the track crosses the meridian of 180°

Long B : 141° 00·0'W

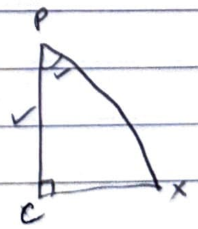
meridian : 180°

P_4, d'long : 39°00'

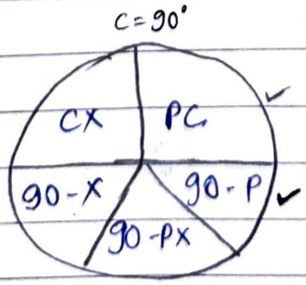
$$P_5 = P_1 - P_4$$

$$= 44^\circ 8{\cdot}8' - 39^\circ 00'$$

$$= 5^\circ 8{\cdot}8'$$



In ΔPCX,



$\cancel{\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})}$

$\cancel{\sin(90-X) = \cos PC \times \cos(90-P)}$

$\cancel{\cos X = \cos PC \times \sin P}$

$\cancel{\cos X = \cos 45^\circ 00' \times \sin 5^\circ 8.8'}$

$\cancel{\cos X = 0.0634}$

$\cancel{X = 86^\circ\ 21.8''}$

$\sin(\text{midpart}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin(90-P) = \tan PC \times \tan(90-PX)$

$\cos P = \tan PC \times \tan(90-PX)$

$$\tan(90-PX) = \frac{\cos P}{\tan PC}$$

$$\tan(90-PX) = \frac{\cos 5^\circ 8.8'}{\tan 45^\circ 00'}$$

$\tan(90-PX) = 0.99596$

$90-PX = 44^\circ 53'$

$PX = 90-44^\circ 53'$

$= 45^\circ 07'$

Latitude $= 90^\circ - 45^\circ 07'$

$= 44^\circ 53'N$

OCT 2024 AM PAPER

Calculate the distance along the composite great circle route from 43° 20'S 146° 30'E to 34° 54'S 073° 15'W so that the vessel does not go south of 45°S. Also calculate the great circle distance.

Soln:

$\angle P$ = d'long

LONG A : 146° 30'E

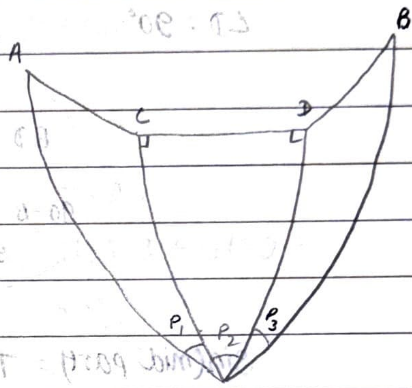
LONG B : 073° 15'W   diff name add

d'long : 140° 15' E

Co, lat, PA = 90° − Lat A

= 90° − 43° 20'

= 46° 40'



Co, lat, PB = 90° − Lat B

= 90° − 34° 54'

= 55° 06'

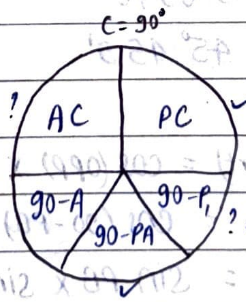
Limiting latitude = 45° S

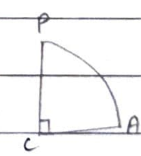
Co-lat, PC or PD = 90° − 45°

= 45°

In ΔPAC

$\angle C = 90°$





sin(mid part) = Tan(adj) x Tan(adj)

$\sin(90 - P_1)$ = Tan(90 − PA) x Tan PC

$\cos P_1$ = Tan(90 − 46° 40') x Tan 45°

$\cos P_1$ = Tan 43° 20' x Tan 45°

$\cos P_1$ = 0.94345

P_1 = 19° 21.6'

$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin AC = \cos(90 - PA) \times \cos(90 - P_1)$

$\sin AC = \sin PA \times \sin P_1$

$\sin AC = \sin 46^\circ 40' \times \sin 19^\circ 21.6'$

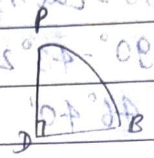
$\sin AC = 0.241126$

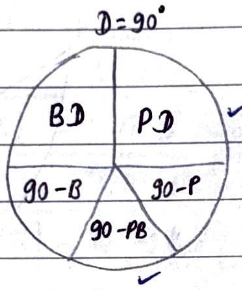
$AC = 13^\circ 57.2'$

$AC = 837.18\ NM$

In ΔPBD,

$\angle D = 90^\circ$





$\sin(\text{mid part}) = \tan(\text{adj}) \times \tan(\text{adj})$

$\sin(90 - P) = \tan(90 - PB) \times \tan PD$

$\cos P_3 = \tan(90 - PB) \times \tan PD$

$\cos P_3 = \tan(90 - 55^\circ 06') \times \tan 45^\circ$

$\cos P_3 = \tan 34^\circ 54' \times \tan 45^\circ$

$\cos P_3 = 0.6976$

$P_3 = 45^\circ 45.9'$

$\sin(\text{mid part}) = \cos(\text{opp}) \times \cos(\text{opp})$

$\sin BD = \cos(90 - PB) \times \cos(90 - P_3)$

$\sin BD = \sin PB \times \sin P_3$

$\sin BD = \sin 55^\circ 06' \times \sin 45^\circ 45.9'$

$\sin BD = 0.5876$

$BD = 35^\circ 59.3'$

$BD = 2159.32\ NM$

Since, $P_1 + P_2 + P_3 = 140^\circ 15'$

$19^\circ 21.6' + P_2 + 45^\circ 45.9' = 140^\circ 15'$

$P_2 = 75^\circ 07.5'$

$$\cos m'lat = \frac{dep}{d'long}$$

$$m'lat = \frac{45+45}{2} = 45^\circ$$

$$d'long = 75^\circ 07.5'$$

$$\cos 45^\circ = \frac{dep}{75^\circ 07.5'}$$

$$dep = 53^\circ 07.3'$$

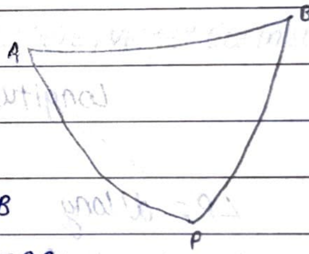
Distance, CD = 3187.3 NM

Total composite G.C distance = AC + CD + BD

= 837.18 + 3187.3 + 2159.32

= 6183.8 NM

To find gc distance



$$\cos P = \frac{\cos AB - \cos PA \cdot \cos PB}{\sin PA \cdot \sin PB}$$

$$\cos P \cdot \sin PA \cdot \sin PB = \cos AB - \cos PA \cdot \cos PB$$

$$\cos AB = \cos P \cdot \sin PA \cdot \sin PB + \cos PA \cdot \cos PB$$

$$\cos AB = \cos 140^\circ 15' \cdot \sin 46^\circ 40' \cdot \sin 55^\circ 06' + \cos 46^\circ 40' \cdot \cos 55^\circ 06'$$

$$\cos AB = -0.066$$

$$AB = 93^\circ 47.1'$$

G.C distance = 5627.15 NM

JAN 2025 PM
APRIL 2023
MAY 2022
JAN 2022

A vessel at TOKYO ($35^\circ 39'N$, $139^\circ 47'E$) intends to sail due east for one day (speed 16 knots, clock advanced by one hour), then on a great circle track to SAN Francisco ($37^\circ 48'N$ $122^\circ 24'W$). Find the maximum latitude arrived during great circle passage.

Sol:- After sailing due east for one day,

$dist = dep = 16 \times 23$

$= 368$ NM

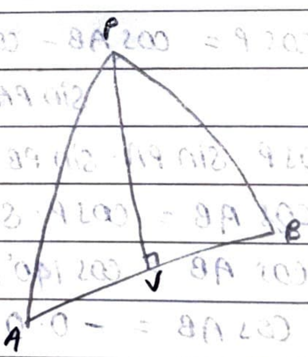
So, $\cos lat = \dfrac{dep}{d'long}$

$\cos 35^\circ 39' = \dfrac{6^\circ 8'}{d'long}$

$d'long = 7^\circ 32{\cdot}9'E$

So, latitude arrived after one day $= 35^\circ 39'N$ (Easterly course)

Initial longitude: $139^\circ 47'E$
d'long: $7^\circ 32{\cdot}9'E$
Longitude arrived: $147^\circ 19{\cdot}9'E$



$\angle P = d'long$

long A $= 147^\circ 19{\cdot}9'E$
long B $= 122^\circ 24'W$ diff name add
$d'long = 90^\circ 16{\cdot}1'E$

co lat, $PA = 90^\circ - lat A$
$= 90^\circ - 35^\circ 39'$
$= 54^\circ 21'$

co lat, $PB = 90^\circ - lat B$
$= 90^\circ - 37^\circ 48'$
$= 52^\circ 12'$

$$\cos P = \frac{\cos AB - \cos PA \cdot \cos PB}{\sin PA \cdot \sin PB}$$

$\cos P \cdot \sin PA \cdot \sin PB = \cos AB - \cos PA \cdot \cos PB$

$\cos AB = \cos P \cdot \sin PA \cdot \sin PB + \cos PA \cdot \cos PB$

$\cos AB = \cos 90°16.1' \cdot \sin 54°21' \cdot \sin 52°12' + \cos 54°21' \cdot \cos 52°12'$

$\cos AB = 0.354$

$AB = 69° \; 15.3'$

$$\cos A = \frac{\cos PB - \cos PA \cdot \cos AB}{\sin PA \cdot \sin AB}$$

$$\cos A = \frac{\cos 52°12' - \cos 54°21' \cdot \cos 69°15.3'}{\sin 54°21' \cdot \sin 69°15.3'}$$

$\cos A = 0.535$

$A = 57°39.8'$

$$\cos B = \frac{\cos PA - \cos PB \cdot \cos AB}{\sin PB \cdot \sin AB}$$

$$\cos B = \frac{\cos 54°21' - \cos 52°12' \cdot \cos 69°15.3'}{\sin 52°12' \cdot \sin 69°15.3'}$$

$\cos B = 0.495$

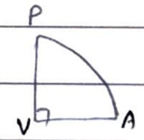
$B = 60°20'$

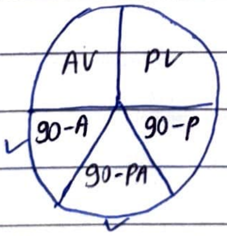
Since, both $\angle A$ & $\angle B$ are acute angle, vertex lies in b/w
both Lat A & Lat B are in same hemisphere, vertex lies in same hemisphere

In ΔPAV,

Using napiers rule,

$\angle V = 90°$





sin(mid part) = cos (opp) X cos (opp)

$\sin PV = \cos(90-A) \times \cos(90-PA)$

$\sin PV = \sin A \times \sin PA$

$\sin PV = \sin 57°39.8' \times \sin 54°21'$

$\sin PV = 0.6866$

$PV = 43° \; 21.6'$

$Lat_V = 46° \; 38.4' N$ i.e. max^m Latitude arrived.