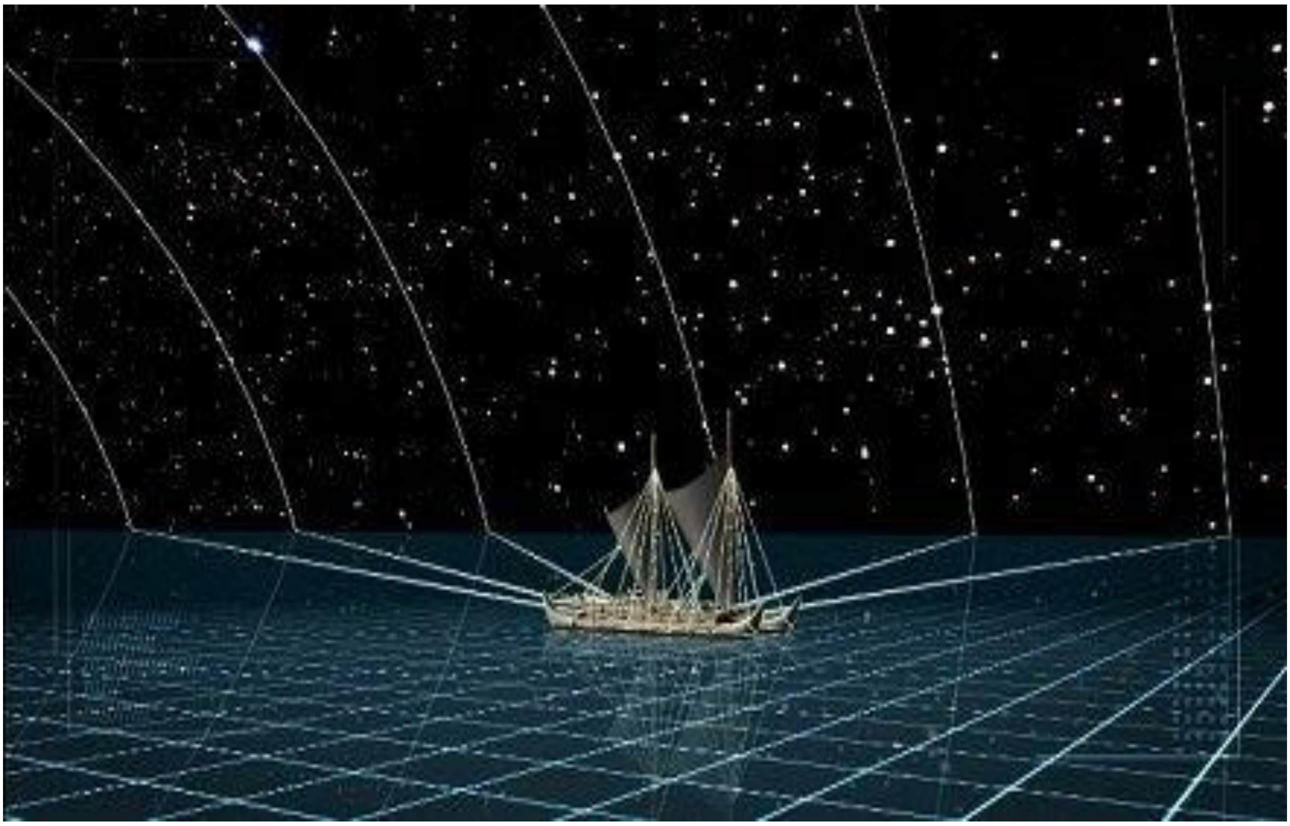
# CELESTIAL NAVIGATION



2nd Mate written notes

PRINCIPLE NAVIGATION

THEORY+ NUMERICAL

By : Anupam Singh Rajput

: SMART MARINER(Please subscribe)

IF YOU HAVE ANY SUGGESTION FOR ME, PLEASE EMAIL IT TO:

anupamsinghrajput8084@gmail.com

# Principle Navigation

No. 135 Date 25/04/2022

## TOPIC No ①:- Nautical astronomy

**Ques①:- Define the following with neat sketches

ⓐ Declination (5 times)

- Declination of a celestial body is the arc of a celestial meridian or the angle at the centre of the Earth contained between the Equinoctial and the parallel of the declination through that body.
- Declination are measured from 0° to 90°, N or S of the equinoctial

(b) Sidereal Hour Angle (SHA) (6 times)

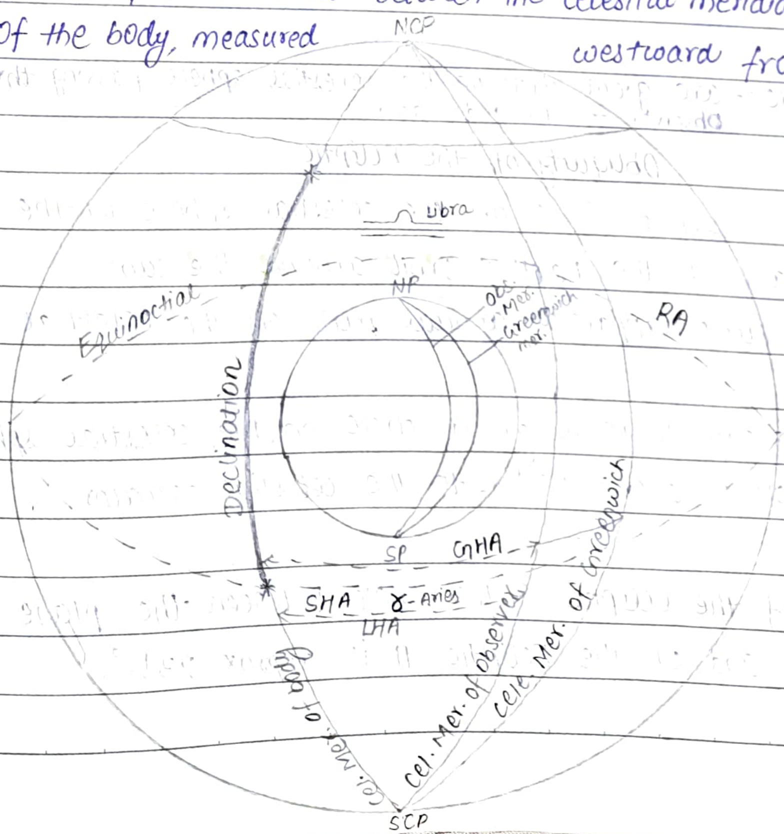
- SHA of a celestial body is the arc of the Equinoctial or the angle at the celestial poles contained between the celestial meridian of the First Point of Aeries and that through the body, measured westward from Aries

(C) Right Ascension (RA) (4 times)

RA of a celestial body is the arc of the Equinoctial or the angle at the celestial poles contained between the celestial meridian of the First Point of Aeries and that through the body, measured eastward from Aries.

(d) Greenwich Hour angle (GHA) (2 times)

GHA of a celestial body is the arc of the Equinoctial or the angle at the celestial poles contained between the celestial meridian of Greenwich and that of the body, measured westward from greenwich.



(e) Local Hour angle (LHA) (3 times)

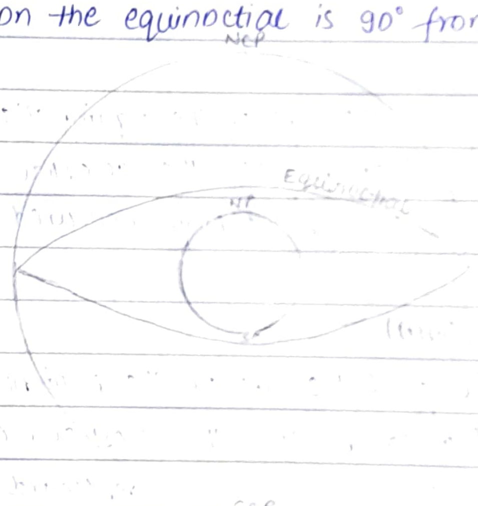
LHA of a celestial body is the arc of the equinoctial or the angle at the celestial poles contained between the observer's celestial meridian and the celestial meridian through that body, measured westward from the observer.

If measured eastward from the [illegible] It is known as Easterly Hour Angle (EHA)

Ques ②: Define the following with neat sketches

@ Equinoctial (Celestial Equator)

- Equinoctial is a great circle on the celestial sphere in the same plane as the plane of the Earth's equator.
- Thus, the equinoctial is the projection of the equator on the celestial sphere.
- Every point on the equinoctial is 90° from the celestial poles

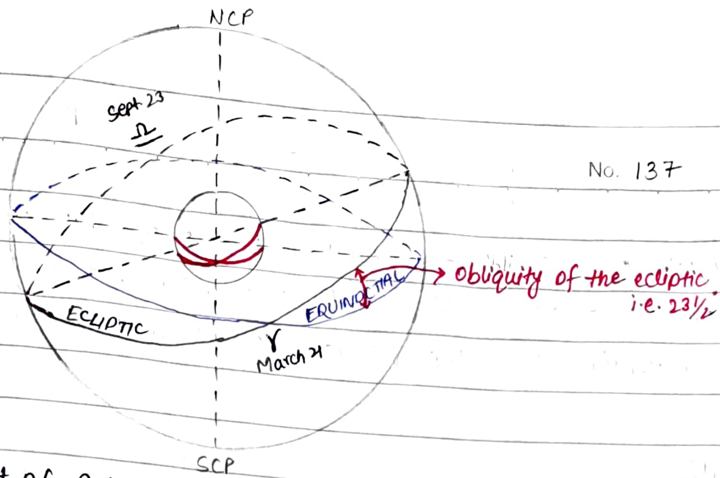



Vertical circle :- are great circle on the celestial sphere passing through the Observer zenith and nadir.

ⓑ Ecliptic (2 times), Obliquity of the Ecliptic

- Ecliptic is a great circle on the celestial sphere in the same plane as the plane of the Earth's orbit around the sun
- Thus, the sun's apparent annual path on the celestial sphere is the Ecliptic.
- Or we can say, It is a great circle on the celestial sphere, inclined at the angle of about 23½° to the celestial equator. This angle is known as obliquity of the ecliptic.

Obliquity of the ecliptic is the angle between the plane of the Equinoctial and of the Ecliptic. It is approx 23½°.




ⓒ First Point of Aries and First point of Libra

On 21st March, at vernal Equinox, the Sun appears to cross the Equinoctial from South to North. This point is known as the First point of Aries. It is denoted by the symbol ♈

On 23rd September, at Autumnal Equinox, the sun appears to cross the Equinoctial from North to South. This point is known as First point of libra. It is denoted by symbol ♎

*Ques③: Define the following with neat sketches

***ⓐ Celestial or Rational Horizon (6 times)

The observer's rational horizon is a great circle on the celestial sphere, which every point is 90 degrees away from the observer's zenith.

ⓑ Visible horizon.

- The line at which the sky and the Earth (sea level) appears to meet
- The horizon which is actually observed at sea.
- This horizon is used as the refrence for celestial observation like sextant altitude or establishing of sunrise & sunset timings.
- The radius of visible horizon increases as the height of eye increases

ⓒ True altitude (2 times)

True altitude of a body is the arc of the vertical circle through that body contained between the rational horizon and the centre of the body.

ⓓ Zenith distance

Zenith distance of a body is the arc of the vertical circle through that body contained between the Observer's zenith and the centre of the body.

Since every point on the rational horizon is 90° from the observer's zenith,

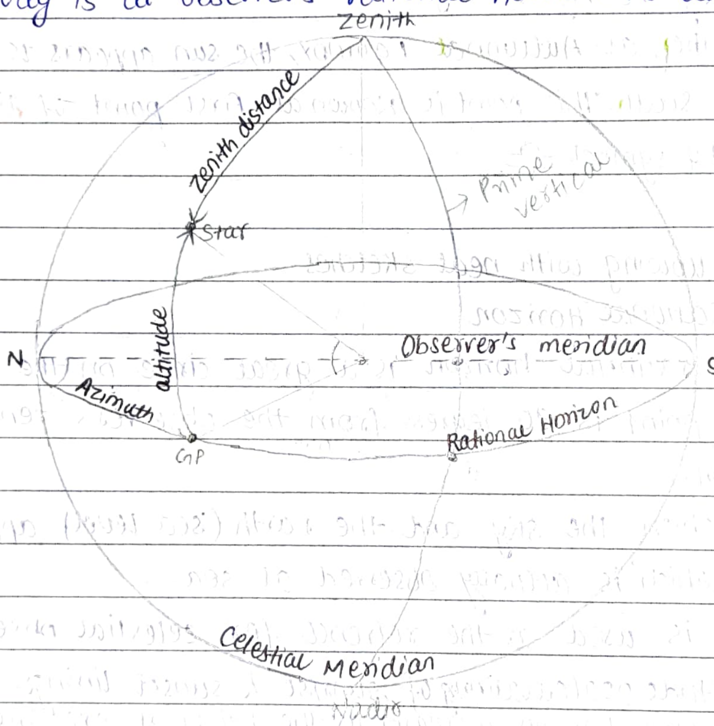
zenith distance = 90° − altitude.

ⓔ Azimuth (3 times)

The azimuth of a celestial body is the arc of the observer's rational horizon or the angle at his zenith, contained between the observer's celestial meridian and the vertical circle through that body.

ⓕ Amplitude (4 times)

The amplitude of a celestial body is the arc of the observer's rational horizon or the angle at his zenith contained between the observer's prime vertical and the vertical circle through that body, when the body is at observer's rational horizon i.e. at sunrise or sunset.




ⓖ Geographical position of a celestial body.

Geographical position of a celestial body is the point on the surface of the earth i.e. the point at which a straight line from the centre of the earth to the celestial body meets the Earth's surface.

ⓗ Prime vertical

The observer's prime vertical is the vertical circle passing through the East and west points of his rational horizon.

Foci is plural of focus. It means point of attraction.

## Topic No(2):- Solar system

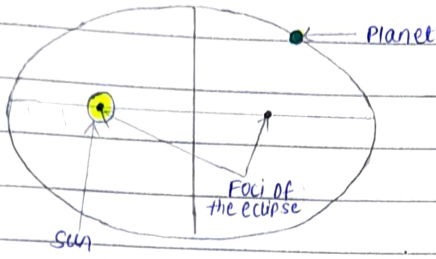
No. 139 Date 26/04/2022

Ques(1): State and explain Kepler's laws of planetary motion (8 times)

Ans:- Kepler's first law states that all planets revolve around the sun in elliptical orbits, with the sun situated at one of the foci of the elliptical orbit.

This law means that the motion of the planets around the sun follows an elliptical path, not a circle.

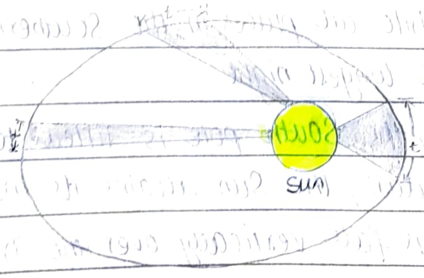
As an ellipse has two foci, according to this law, the sun is considered to be situated at one of the foci




Kepler's second law states that each planet revolves around the sun in such a way that the line joining the planet to the sun sweeps the equal areas in equal interval of time.

In its orbit, the point at which the planet is farthest from the sun is known as aphelion & the point at which the planet is nearest to the sun is known as perihelion.

So, this law basically says that the planet speed is not constant - it travels slower when it is farthest from the sun (aphelion) & travels faster when it is nearest to the sun (perihelion).

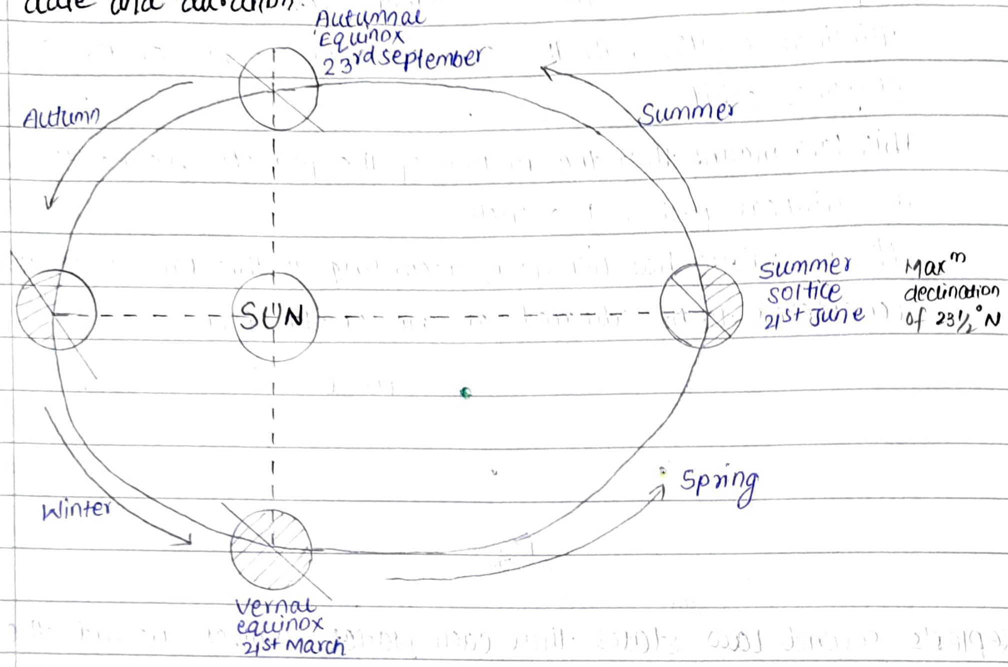



Kepler's third law gives the relationship between the distance of a planet from the sun and the time it takes to complete one revolu-tion around the sun.

The square of the orbital periods of the planet is directly propo-rtional to the cube of its average distance from the sun.

This law says that, planets which are closer to the sun have greater angular velocity than the planet which are farther away.

***Ques⑤:- Explain with suitable sketches how are seasons caused giving appropriate date and duration.




The earth spin axis is tilted with respect to its orbital plane. This is what causes the seasons.

Let us consider the earth at four important points in its orbit.

⇒ • On 21st June, the North pole is tilted towards the sun by maximum of 23½°. On this date, the Sun attains its maximum declination North and the sun's rays fall vertically over the Tropic of Cancer. The sun is then said to be at Summer solstice.

• All places in the Northern Hemisphere then have the longest day and shortest night while all places in the Southern Hemisphere have the shortest day and longest night.

⇒ • On 22nd December, the South pole is tilted towards the sun by maximum of 23½°. On this date, the Sun attains its maximum declination South and the sun's rays fall vertically over the Tropic of Capricorn. The sun is then said to be at Winter Solstice.

• All places in the Southern Hemisphere then have the longest day and shortest night while all places in the Northern Hemisphere have the shortest day and longest night.

⇒ • On 21st March, the Earth axis is tilted in a direction at right angle to the direction from Earth to Sun. On this date, the sun's declination is 0° and the sun's rays fall vertically over the Equator. The sun is then said to be at **vernal equinox**

- All the places on earth have equal days and night of 12 hours duration each. The sun rises at 6 AM and sets at 6PM throughout the world.

→ On 23rd September, the Earth axis is again tilted in a direction at right angle to the direction from Earth to sun. On this date, the sun's declination is 0° and the sun's rays again fall vertically over the Equator. The sun is then said to be at **Autumnal equinox**.

- All the places on the earth again have equal days and nights of 12 hours duration each. The sun once again rises at 6 AM and sets at 6 PM throughout the world.

→ From vernal equinox to summer solstice, when the sun's declination is increasing from 0° to its $max^m$ value of 23½°N, the Northern Hemisphere is said to have **spring** season.

→ From summer solstice to Autumnal equinox, when the sun's declination decreases from (maxm of) 23½°N to 0°, the Northern Hemisphere is said to have **summer** season.

→ From Autumnal equinox to Winter solstice, when the sun's declination increases from 0° to its $max^m$ value of 23½°S, the Northern Hemisphere is said to have **autumn** season.

→ From winter solstice to vernal equinox, when the sun's declination decreases from 23½°S to 0°, the Northern Hemisphere is said to have **winter** season.

It should be noted that the 4 seasons are not of equal lengths. The varying speed of earth in its orbit causes the season of unequal lengths, approximately as follows:- Spring : 93 days
Summer : 94 days
Autumn : 90 days
Winter : 89 days.

| | | |
|---|---|---|
| V | 21 | march |
| S | 21 | June |
| A | 23 | september |
| W | 22 | december |

Ques 6:- Define (or briefly explain) the following :-

(a) Elongation

It is the angle at the centre of the Earth contained between the centre of the Sun and the centre of the planet or moon, measured along the plane of the Ecliptic.

A planet can be at an eastern or western elongation, depending on whether the planet lies to east or west of the sun as seen from the Earth.

The elongation of superior planet can vary from 0° to 180° & the elongation of inferior planets range between 0° and a greatest elongation.

(b) Conjuction

- A planet or the Moon is said to be in conjuction with the Sun, when it is in same direction as the sun, when seen from the Earth (i.e their celestial longitudes are same)
- An elongation is of 0°
- An inferior conjuction occurs when the planet lies between the Earth & the sun. A superior conjuction occurs when the planet lies on the opposite side of the sun.

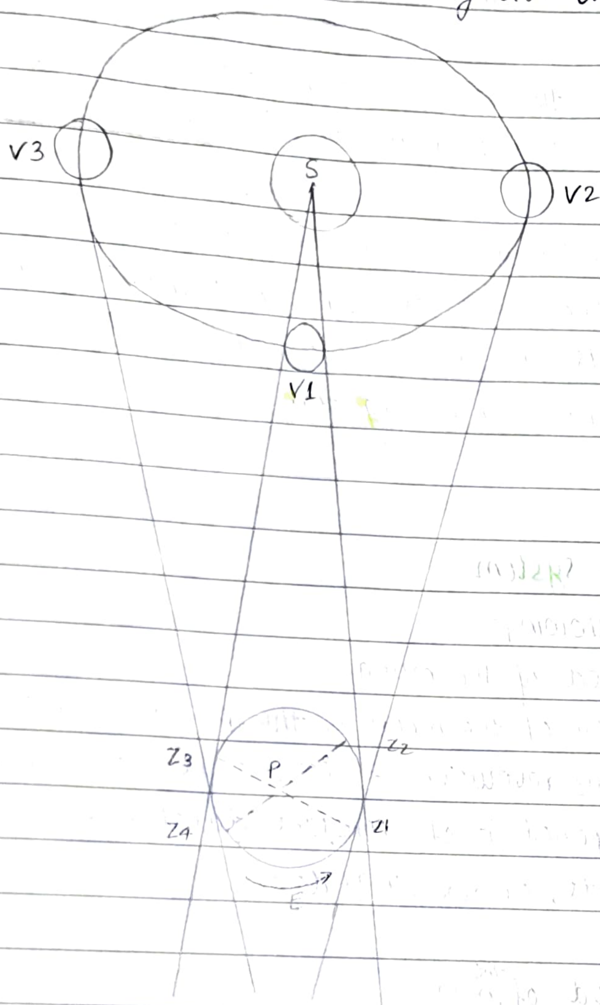
(c) Opposition.

- A planet or the Moon is said to be in opposition with the Sun, when it is in opposite direction to the sun, when seen from the Earth (i.e. their celestial longitudes are 180° apart)
- An elongation is of 180°
- Inferior planets can never be at opposition.

(d) Quadrature.

- A planet or the Moon is said to be in quadrature when its elongation is exactly 90° East or 90° west. It depends on whether the planet lies to the east or west of the Sun, when seen from the Earth.
- As elongation is of 90°
- Inferior planets can never be at quadrature.

Explain why venus is called a morning star or a evening star.




The figure shows the sun, the Earth and venus at three positions in its orbit.

→ At V1 :- When the venus is in conjuction with the sun, when seen from the Earth, they would appear to rise, culminate and set together.

→ At V2 :- • When the venus is on the east side of the earth in its orbit, a person on the earth at Z1 would see venus rising. The sun would still be below his horizon. For the sun to rise, the earth would have to rotate further by some degrees, till the observer is brought to position Z2.

• Therefore, venus would be visible on eastern horizon for few hours before sunrise. Once the sun rises, due to its brilliance, venus would not be visible during the day.

• Since, the venus had risen before the sun, it would also set before the sun and therefore not be visible in the evening after sunset.

Therefore, it is called a morning star.

→ At V3 :- • When the venus is on the west side of the earth in its orbit, a person on the earth at Z3, would see the sunset. The venus would still be above the horizon. For the venus to set, the earth would have to rotate further by some degrees, till the observer is brought to position Z4.

- Therefore, venus would be visible on the western horizon for few hours after sunset.
- Since the venus had set after the sun, it would rise after the sunrise on the next day and therefore not be visible in the morning. Once the sun rises, due to its brilliance, venus would not be visible during the day. Therefore, it is called a evening star.

## Topic ③:- Earth-Moon system

Ques ⑧:- Define the following:-

*ⓐ Sidereal period of the moon.

- Sidereal period of the moon is the period of time taken by the Moon to complete one revolution of 360° around the earth.
- The sidereal period is of constant duration i.e. 27 days 07 hrs 43 mins. and 12 seconds, approx. 27.33 days.

ⓑ Synodic period of the moon. (lunar month)

- Synodic period of the moon is the period of time between two consecutive full moons or new moons.
- The synodic period is of average duration i.e. 29 days 12 hours 44 mins.
- The synodic period is also known as "lunar month", a "lunation" or a "Synodic Month".
- The duration of synodic period is not constant. It can have variation of upto 13 hours due to eccentricity of the moon's orbit around the earth.

ⓒ Age of moon.

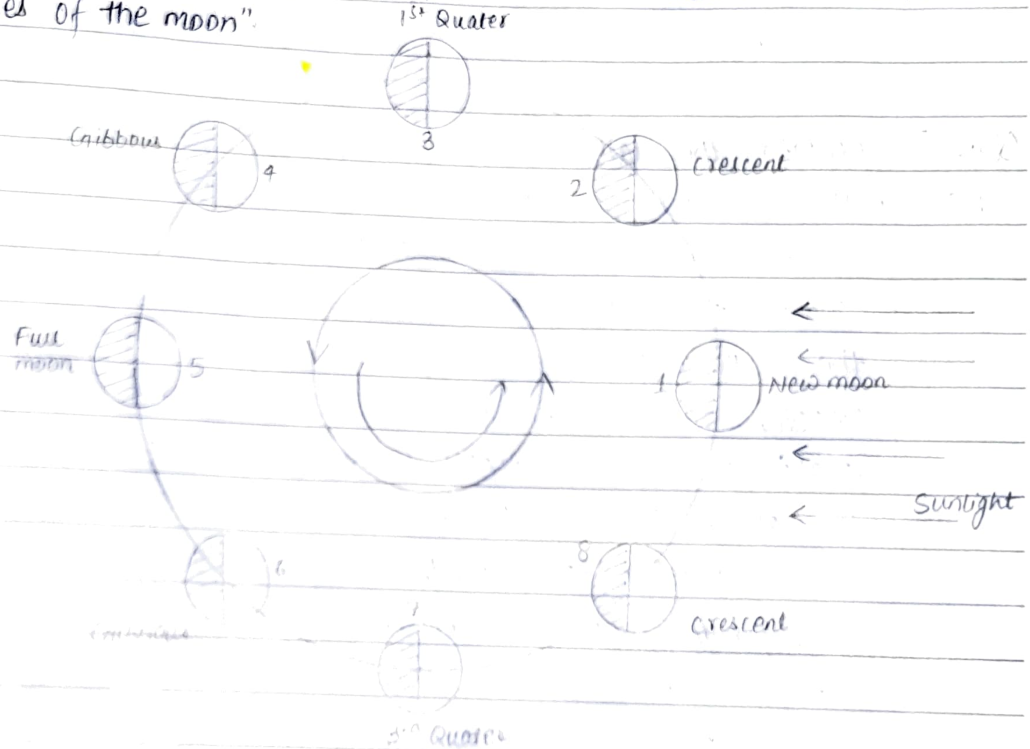
- It is a way of expressing its phase.
- It represents the number of days since the last new moon.

Explain various Phases of the moon (2 time)

The moon is not self luminuos. We see the moon, as it reflects the sunlight. Being spherical, 50% of the moon's surface area is always illuminated by the sun.

The amount of Moon's illuminated area visible from earth, varies with the relative position of the Sun and moon with respect to Earth.

The various shapes of moon visible from the earth due to this is termed as "Phases of the moon".




(1) When the moon is in conjuction, its entire illuminated hemisphere is turned away from earth. No part of illuminated surface is visible from earth & moon is then said to be "New Moon"

(2) As the moon moves in its orbit, a small part of the moon's illuminated surface is visible from earth in the form of crescent

(3) About 7½ days from the New moon, when the moon is in quadrature, exactly half of the moon's illuminated surface is visible from the earth. This is "First Quater" of the moon

(4) As the moon moves further in its orbit, more than half of the moon's illuminated surface is visible from earth, which is termed as "gibbous"

(5) After 14-15 days from the New moon, the moon comes in opposition of the Sun. The entire moon's illuminated surface faces the earth & hence entire disc is visible from the earth & the moon is then said to be "Full moon".

Culminate : end up, completion, अंत
Conjuction : combination, addition, संयोजन, गुटबंदी



**Posn 6:** During the second half of synodic period of the moon, the illuminated surface of the moon visible from earth decreases & moon appear "gibbous" again.

**Posn 7:** After 22 days from the New moon, again exactly half of the moon's illuminated surface is visible from Earth. This is "third quater" or last quater of the moon

**Posn 8:** The moon again form a crescent shaped & finally returns to "New moon". The average duration of this cycle is 29½ days.

Earth rotates 360° in 24 hours
15° — 1 hours

since the moon had moved eastward by 12.2°, so to culminate, Earth had to rotate further 12.2° that take
12.2° = $\frac{1}{15} \times 12.2°$ = 49 min

**Ques 10:** Explain the daily retardation of the moon.

**Ans:-**

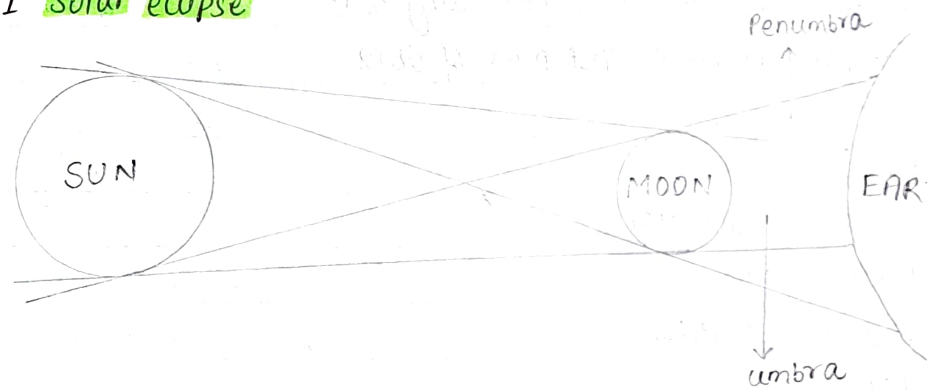
- At new moon, when the sun and moon are in conjuction, they would culminate at the same time. During the course of one day, the moon would have moved eastward by 360°/29½ = 12.2° in its orbit around the earth, with respect to the sun.
  (synodic period is the time between two consecutive full moon or new moon.)
- Exactly one day after new moon, when the earth has completed one rotation of 360° with respect to sun, the sun once again culminates.
- But as the the moon has moved eastward by 12.2° & since the earth rotates at 15° per hour, earth would have to rotate further for approx 49 mins. for the moon to culminate again.

Thus, the moon culminates about 50 mins later each day. If the declination of the Moon remain unchanged, it would also rise and set approx. 50 min later each day.

This phenomenon is known as Daily retardation of the moon.

**Ques 11:** With the help of suitable diagrams, explain solar eclipse.

7.5.1 Solar eclipse




- When a moon is in conjuction with the sun & the centre of the

No. 147 Date 28/04/2022

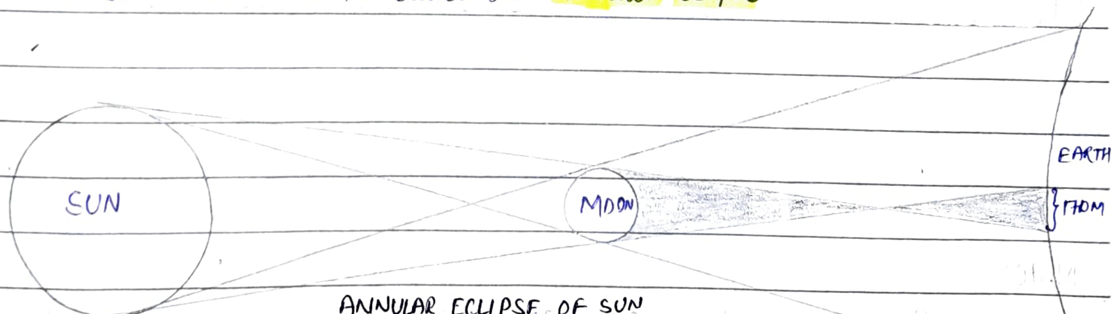
three bodies (sun, moon & earth) are neary in a line, the Moon appears over the sun & (blocks the sun's disc, wholly or partly) or blocks the view of sun from the earth. Such an occurence is called a Solar eclipse.

- The shadow cast by the moon is conical in shape.
  The shadow cone within which no light of the sun reaches is called Umbra.
  The widening cone shaped region around the umbra, where part of the sun's light reaches is called Penumbra.
- There are three types of solar eclipse

(a) Total eclipse: People on the earth within the area where umbra cone of the moon falls, where no light from the sun reaches, will have total darkness because the moon covers the entire face of the sun. This occurence is termed "Total eclipse".

(b) Partial eclipse: People on the earth outside the umbra region, but within the penumbra region, where part of the sun light reaches and remainder covered by the moon. Such occurence is termed "Partial eclipse".

(c) Annular eclipse: • (As the orbit of the moon around the earth is elliptical & ecentric) when the moon is near apogee (i.e. when it is farthest away from the earth in its elliptical orbit), it can happen that umbra cone of moon does not reach the earth surface.
- People on the earth see the sun with the moon obscuring the central portion of the Sun's disc.
  Such occurence is called an "Annular eclipse".




ANNULAR ECLIPSE OF SUN

Note:-
- The maximum diameter of the area on earth, over which umbra cone falls is about 170 miles
- The maximum diameter of the area on earth, over which penumbra region falls is about 4000 miles.
- A period of total/annular eclipse can never exceed about 8 mins at any one time.
- A solar eclipse can take place only on a new moon day but not necessary that it will take place on each new moon day.
- A total or annular eclipse starts & end with partial eclipse

Smart Mariner Edition
for smart sailors

No. 148 Date 28/04/2022

**

Ques ⑫:- ⓐ Explain with aid of suitable sketches, what is lunar eclipse?

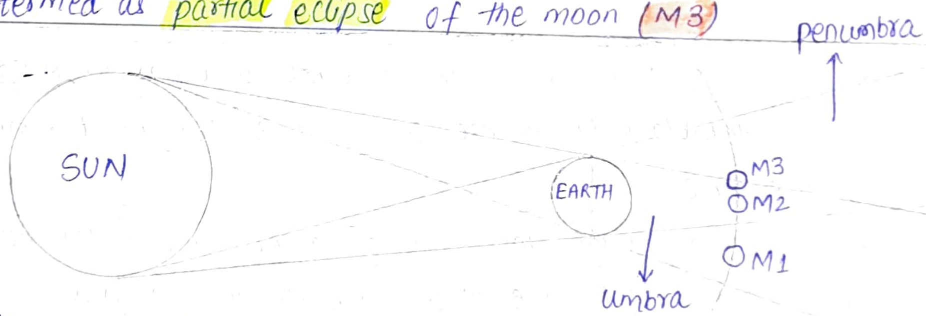
Ans:- The earth have a shadow behind itself. The shadow consist of a coneshaped, tapering umbra, where no light from the sun reaches & widening penumbra region, where some sunlight does reach. Since, the moon is not self luminuos & we see it only when it reflects a sunlight. A lunar eclipse therefore take place when moon passes through the earth's shadow.

There are three types of lunar eclipse:

ⓐ Total eclipse : When the moon is entirely within the umbra part of earth, where no light from sun reaches any part of the moon. The entire moon then becomes invisible. Such an occurence is termed "Total eclipse" of the moon. (M2)

ⓑ Penumbral eclipse :- When the moon is entirely within the penumbra part of Earth, where part of the sun's rays falls over the entire surface of the moon. We then see the full moon with good briliance. such an occurence is termed as penumbral eclipse (M1)

ⓒ Partial eclipse :- When the moon is partly within the "umbra" & partly within the "penumbra" of the earth, then part of moon within the umbra becomes invisible & part of moon within the penumbra will be visible with good briliance. Such occurence is termed as partial eclipse of the moon (M3)




Note:- • Since the moon is in opposition of the sun, for lunar eclipse to occur, it can take place only in a full moon day & only when moon is or near the ecliptic.

• As the shadow of the earth, must fall on the moon for a lunar eclipse to occur, SHA & GHA should differ by 180°

• Declination of sun & moon should be nearly equal but of opposite names.

• Lunar eclipse need not take place on all full moon days

(b) Why lunar eclipse may not take place on every full moon day?

Ans:- • A lunar eclipse need not take place on every full moon days, because, though the condition regarding their SHA or GHA is satisfied on each Full moon day, but the condition regarding their declination may not be simultaneously satisfied, as the moon's orbit is inclined to the plane of the ecliptic.

• A lunar eclipse will take place only if the Moon is on or near the ecliptic i.e. at or near it nodes on Full Moon day.

## Topic NO. ④ :- TIME

Ques ⑬ :- Define the following:-

ⓐ Sidereal day (2 times)

• It is the interval in time between two succesive meridian passages of First Point of Aries over the same meridian.

• The sidereal day is the true rotational period of the Earth & is of constant duration.

• It has a duration of 23 hours, 56 minutes, 4.1 seconds of mean solar time.

• It is not used as a unit for measurement of time in civil life because life on earth is governed by sun & a solar day is about 4 minutes longer than a sidereal day.

ⓑ Equation of time (3 times)

• Equation of time is the difference between the Mean time and the Apparent time, measured from the same meridian.

• It is expressed as Equation of time = Mean time − apparent time. by navigators

• It is expressed in minutes and seconds of time.

• If mean time is greater than Apparent time, equation of time is +ve and if apparent time is greater than mean time, equation of time is −ve.

• Equation of time values are tabulated in the daily pages of the nautical alamanac for 00 hrs and 12 hrs GMT on each day.

**Ques No (14):- What is International Date Line? Why is it necessary and how is the date on a ships crossing IDL on an Easterly course affected? (3 times)**

Ans:-
- The International Date line (IDL) is an imaginary line running between South Pole and North Pole defining the boundary between one day & next.
- It passes through the Pacific ocean, roughly following the 180° line of longitude and it deviates little to pass around some territories and island groups.
- It is located halfway around the world from the prime meridian (0° longitude line in Greenwich, England)

It is necessary because

- We know that, if a ship proceeds eastward, she would have to advance her clock by one hours for every 15° of d'long & if a ship proceeding westward, she would have to retard her clock by one hour for every 15° of d'long, to indicate the correct LMT.
- Let us consider, a ship navigating eastward making a d'long of 12° per day. She would return to her original (from where she started) meridian in 360/12 = 30 days. During this period, she would have to advance her clock by one hours for every 15° of d'long i.e total of 360/15 = 24 hours, or one day. That means, by her calendar, she would return to the original meridian in 31 days.

Similarly, a ship navigating westward making a d'long of 12° per day. She would return to her original meridian in 360/12 = 30 days. During this period she would have to retard her clock by one hours for every 15° of d'long i.e total of 360/15 = 24 hours, or one day. That means, by her calendar, she would return to her original meridian in 29 days.

- Thus, in compare with the date at shore station on original meridian, the date of the ship which sailed eastward is one day ahead & the date of the ship which sailed westward is one day behind.
- To avoid this anamalous situation, the Date line has been introduced by International agreement.

To avoid this

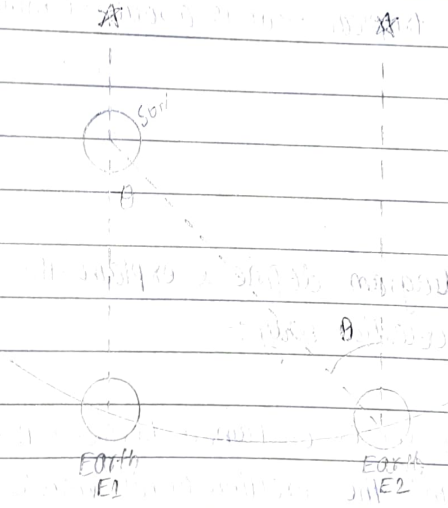
Ships crossing the IDL, on an easterly course, have to retard their date by one day & ships crossing the IDL, on an westerly course, have to advance their date by one day.

Ques No (15):- Explain why star culminate/ rise or set about 4 minutes earlier each day? (4 times)

Ans:-
- This happens because of the revolution and rotation of the earth.
- So, there is a "solar day" and a "sidereal day"
- We know that, the earth take 23 hours 56 minutes 4.1 seconds (i.e sidereal day true rotational period of sun) to complete one rotation around its axis.
- And, the Earth take 365 1/4 days to complete a revolution of 360° around the sun. So each day, earth move about (360°/365¼ =) approx 1° around the sun in its orbit.
- By the time, the earth complete one rotation, it also moved about 1° in its orbit, which make difference in the direction to the sun
- So now, the earth have to rotate 1° more i.e total of 361° for sun to culminate again, which take further 4 more minutes. 360° = 24 hrs, 15° = 1 hrs = 60 min, 1° = 60/15 = 4 min

  Thus the solar day is about 4 minutes longer than sidereal period i.e 24 hours
- Since, the star is in the same place with respect to earth at all time, no matter where we are in our orbit, so, it takes just 23 hour 56 mins for star to culminate again i.e the sidereal day.

  Hence, the star appears to culminate about 4 mins earlier each day.
- Therefore, stars also rise and sets about 4 minutes earlier each day.

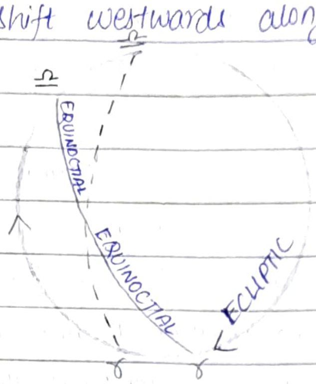



Ques No (16):- Explain with diagram - "Precession of Equinoxes"? How it caused and what are its effects? (2 times)

Ans:- The slow westward motion of the Equinoctial points (First point of Aries and First point of Libra) along the Ecliptic by about 50.2" of arc each year is termed as Precession of Equinoxes. A full cycle of precession occupies about 25,800 years.

It is caused by :-

- As the direction of the Earth's axis changes due to precession, the plane of the equator and the plane of the Equinoctial shifts.
- Since, the plane of the Ecliptic is unchanged, the change in the plane of equinoctial causes the point at which the Equinoctial inter.. the Ecliptic to shift westwards along the Ecliptic




Effects of Precession :

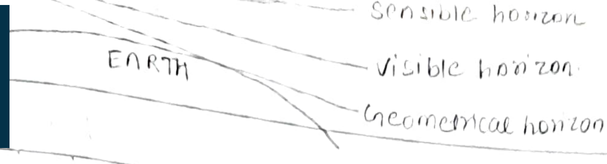
- As the First point of Aries moves westward, the RA of fixes bodies like stars, increases by a corresponding amount, yearly.
- As the plane of the Equinoctial shifts, the declination of stars changes, as declination are measured N or S from the Equinoctial.
- Due to precession, the tropical year is about 20 mins shorter than a sidereal year.

Topic ⑤ :- Altitude

Ques No. ⑰ :- With the help of diagram, define & explain the following correction to be applied to the celestial body :-

ⓐ Dip (3 times)

- It is the angle at the observer contained between the plane of observer's sensible horizon and the direction to his visible horizon
- It occurs because the observer is not situated at sea level
- The value of dip increases as the observer's height of eye increases.
- The values of dip are tabulated on the cover page of the nautical alamanac.
- The value of dip is always '-ve'
- Dip is applied to the observed altitude to obtain the altitude of the body above the sensible horizon which is known as apparent altitude




No. 153 Date 28/01/2022

ⓑ Refraction

- Refraction is the deviation of light rays passing from one medium to another.
- It is the bending of light rays as they travel through succesive layers of various densities of air.
- When passing from a rarer medium into a denser medium, the ray refracted towards the normal to the surface of seperation between the two media.
- Refraction increases the apparent altitude of the celestial body.
- Refraction correction is always negative.
- Refraction has a maximum value of 34.5' when the body is on horizon and it decreases as the altitude increases.

ⓒ Parallax (4 times)

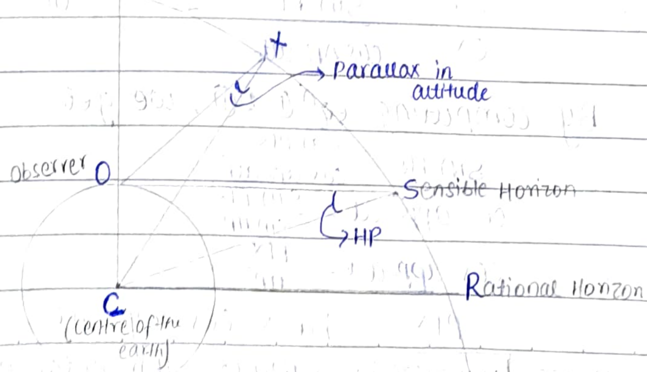
- Parallax is the corrections needed to apply if the observed body is sun, moon or planet.
- This correction is applied to obtain the altitude of the body above the Observer's rational horizon

**

Ques 18: ⓐ Define horizontal parallax & parallax in altitude (5 times)
ⓑ Draw a sketch & show why this correction is always positive.

Ans:- **Horizontal Parallax:** HP of a body is the angle at the centre of the body contained between the centre of the Earth and observer at the surface of the earth, when the body is on the observer's sensible horizon.

**Parallax in altitude:** Parallax in altitude is the angle at the centre of the body contained between the centre of the Earth and observer at the surface of the Earth, when the body is at any any altitude.



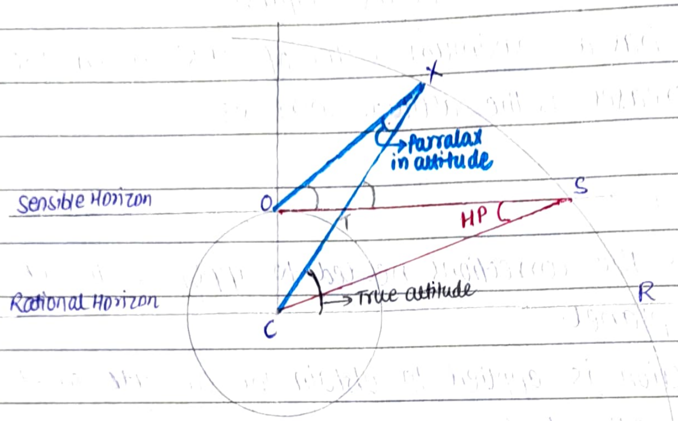
- From figure, ∠SOX is the altitude of the body above the sensible horizon obtained after applying various corrections other than parallax.
- ∠RCX is the altitude of the body above the rational horizon.
- True altitude (∠RCX) = ∠XTS

∠RCX = ∠SOX + ∠OXC

Thus True altitude = altitude above sensible horizon + parallax in altitude.
Therefore, parallax correction is always additive/positive.

ⓒ Prove "Parallax in Altitude = Horizontal parallax × cos app. alt" (3 times)

Ans:-




In (△COS) COS, sin HP = CO/CS

Hence, sin HP = CO/CX (radius) ——— ①

In △COX COX, by sin rule

$$\frac{CO}{\sin PIx} = \frac{CX}{\sin COX}$$

$$\frac{CO}{CX} = \frac{\sin PIx}{\sin COX}$$

$$\frac{CO}{CX} = \frac{\sin PIX}{\sin(90 + App.alt)}$$

$$\frac{CO}{CX} = \frac{\sin PIX}{\cos App.alt} \quad \text{——— (ii)}$$

By computing eqⁿ ① & (ii), we get

$$\sin HP = \frac{\sin PIX}{\cos app.alt}$$

$$\cos app.alt = \frac{\sin PIX}{\sin HP}$$

$$\cos app.alt = \frac{PIX}{HP}$$ (bcz sin of small angle is angle itself)

$$PIX = HP \times \cos app.alt$$

Hence, **Parallel in altitude = Horizontal parallax × cos app. alt**

proved.

# Topic No. 6 :- Nautical Almanac

Ques(1):- Explain the 'v' and 'd' correction given in Nautical Almanac. Why is 'v' correction applied for only certain celestial body? (3 times)
What is 'v' correction? Why it must be applied for planets and moon

Ans:⇒ "V" correction

- The "v" is an extra correction for additional longitude movement of the celestial body.
- The value of "v" correction for any duration of (minutes and seconds) of time can be obtained from increment and correction table of Nautical Almanac
- The "v" correction will always be positive except for venus for which correction may be negative occasionally.
- The "v" corrected tabulated in Nautical Almanac is to correct GHA of the moon and planets.

It is applied only for moon and planets only because:

- This correction is not needed for sun and stars because they circle the earth at constant rate.
- This correction is not needed for sun and stars because its rate of increase of GHA per hour is always nearly equal to 15°.
- Any small difference can be managed in next tabulated hourly value of sun's GHA.
- This correction is needed for moons and planets because they do not circle the earth at constant rate due to its orbital motions.

⇒ "d" correction

- The "d" is an extra correction for additional declination movement of the celestial body. It is the hourly change in the declination of various bodies.
- The value of "d" correction for any duration of minutes and seconds of time can be obtained from increment and correction table of Nautical Almanac.
- The sign of "d" correction depends on whether it is an increase or a decrease.
- "d" is not tabulated for aries, as it is always on Equinoctial, with a constant NIL declination.
- For the sun & planet, the mean value of hourly change of declination for three days, is given at bottom of the table.
- For moon, it is tabulated hourly due to high rate of change of declination.

V.V.I

No. 156 Date 29/04/2...

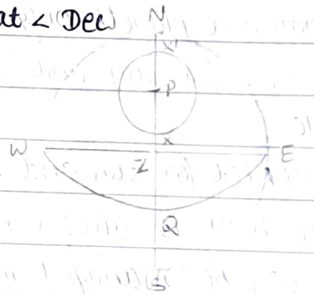
## Topic (7):- Circumpolar

**Ques (20):-** What do you understand by circumpolar body? What are the conditions necessary for a heavenly body to be circumpolar? Substantiate your answer with a suitable sketch. (10 times)

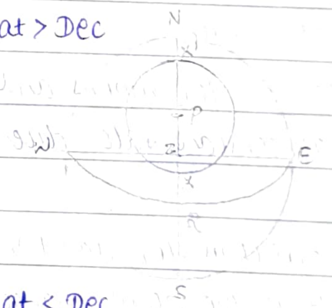
**Ans:-**
- "Circumpolar body" means it is continuosly visible above the horizon. It never goes below the horizon at particular latitude
- For the body to be circumpolar, the body should always be above the rational horizon i.e. body should not set
- Therefore, a circumpolar body will have upper transit (upper meridian passage) which is above the elevated pole & will have lower transit (lower meridian passage) which is below the elevated pole

The condition required for a heavenly body to be circumpolar:
- Dec. > 90° − Lat (For the body not to set) [Lat + dec ≥ 90°]
- Lat & dec should be of same name.

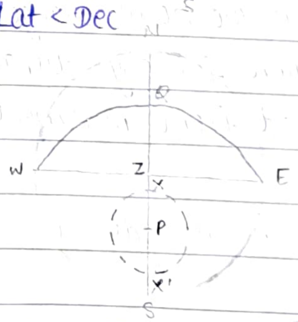
(1) North Latitude → Lat < Dec



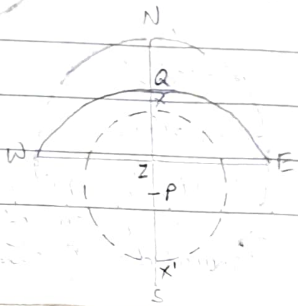

(2) North Latitude − Lat > Dec




(3) South Latitude − Lat < Dec




(4) South Latitude − Lat > Dec



If ...20 is asked for 15 marks, then write more ↓ No. 157 Date 29/04/2022

## Upper and lower transit of circumpolar body

- Upper transit (NX or SX) of a circumpolar body is above the elevated pole

For North latitude,

upper transit bearing is North, when lat < dec (90° − ZX)

upper transit bearing is south, when lat > dec [180° − (XX' + NX')]

For South latitude,

upper transit bearing is south, when lat < dec (90° − ZX)

upper transit bearing is North, when lat > dec 180° − (XX' + NX')

- Lower transit (NX' or SX') of a circumpolar body is below the elevated pole.

For North latitude,

lower transit (NX') bearing is North, same as elevated pole or latitude or declination = 90° − (PX' + PZ)

For South latitude,

lower transit (SX') bearing is south, same as elevated pole or latitude or declination 90° − (PX' + PZ)

## Topic No (8): Amplitude

Ques (21): What are the condition necessary for a heavenly body to cross prime vertical? (3 times) b) Rise bearing true east / setting bearing true west

Ans: a) condition necessary for a heavenly body crossing a prime vertical :-

- It bears true east or true west when it is at right angle to the meridian.

b) A condition necessary for a heavenly body rising true east or setting true west

- The declination should be zero.

* If a circumpolar bodies crossing a prime vertical, condition necessary are:-

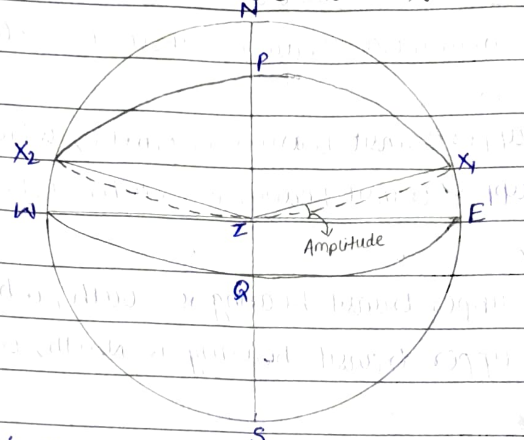
- Lat + dec ≥ 90°
- Lat & dec should be of same name
- Lat > dec

No. 158 Date 29/04/2022

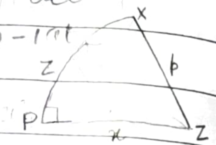
**

Ques (22) Prove sin Amplitude = sin Declination x sec latitude (8 times)

Ans:-

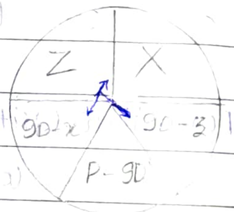



From PZX triangle,
PX = 90 - dec
PZ = 90 - lat




Side p = 90°
will go clockwise

∠Z x ∠P z ∠X
90-x 90-P 90-z




Using Napier's Rule, sin (mid part) = cos (opposite) x cos (opposite)

In quadrantal spherical Δ PZX

~~$\sin(90-z) = \cos(90-x) \cdot \cos Z$~~

$\sin(90-PX) = \cos(90-PZ) \cdot \cos Z$

~~$\sin[90-(90-dec)] = \cos[90-(90-lat)] \times \sin(90-Z)$~~

$\sin dec = \cos lat \times \sin(90-Z)$

$\sin dec = \cos lat \times \sin Amp.$

$\sin Amp = \sin dec / \cos lat$

$\sin Amp = \sin dec \times \frac{1}{\cos lat}$

$\sin Amp = \sin dec \times \sec Lat$

Hence, Proved

## TOPIC (9):- Rising - setting of celestial body.

Ques (23):- Prove that the altitude of celestial pole is equal to the latitude of the observer (1 times)

Not imp

Ans:- Let 'O' is the observer at celestial sphere

Let 'P' is the position of the North celestial pole (NCP) w.r.t to 'O'

let 'a' denote the altitude of celestial pole ("P")

Let 'ϕ' denote the latitude of observer

To proof :- $a = \phi$

Proof :- Let 'Z' denote the zenith distance of celestial pole, 'P'

let θ denote the colatitude of observer (connect obs. posn to pole posn)

By definition, Altitude, $a = 90° -$ zenith distance $(z)$

Latitude, $\phi = 90° -$ colatitude $(\theta)$

Since, zenith distance of NCP is equal to colatitude of observer

$z = \theta$

$90° - z = 90° - \theta$

$a = \phi$ Hence proved

Ques 4 :- Explain the difference between theoretical sunrise/sunset & visible sunrise/sunset (5 times)

Ans:- Theoretical sunrise/sunset

- It occurs when the true sun's centre is on the observer's rational horizon.
- Then, the true altitude of sun is 0° and the true zenith distance is 90°.
- The time of theoretical sunrise/sunset can be obtained by solving PZX triangle, where ZX is 90°

Visible sunrise/sunset

- At visible sunrise/sunset, the true altitude of sun is not 0° because of corrections for refractions, semi-diameter, dip etc.
- Assuming the observer at sea level the true altitude of sun at visible sunrise/sunset is 0° 50′ & the true zenith distance is 90° 50′.
- Because of this, visible sunrise occurs before theoretical sunrise & visible sunset occurs after theoretical sunset.
- The time of visible sunrise/sunset can be obtained from Nautical Alamanac provided for various latitudes. Interpolation is required for intermediate latitude.

Ques 5 :- Explain the factor which governs the period of daylight for an observer.

Ans:-

- Our amount of daylight hours depends on our latitude and how Earth orbits the sun
- Earth's axis of rotation is tilted from its orbital plane. As a result, the orientation of Earth's axis to the sun is always changing throughout the year as we revolves around the sun.
- As this orientation changes, so does the distribution of sunlight on Earth's surface changes at any given latitude.
- The intensity of sunlight reaching the earth's surface governs the number of hours of daylight.
- This tilting of earth spin axis also causes the seasonal variation.

Topic No. 10 :- Twilight



*** Ques (26) :- Define twilight. Explain clearly the cause of twilight and the reason why twilight lasts longer in higher latitudes. (6 times)

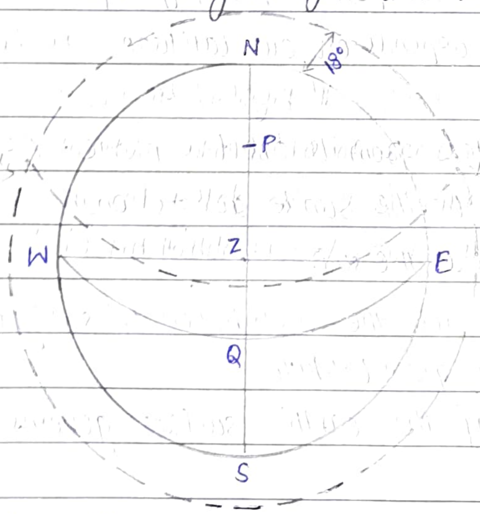
Ans :-
- Twilight is the light recieved from the sun, when the sun is below the horizon, that is before sunrise in the morning and after sunset in the evening.
- It is the time between day and night when the sun is below the horizon but its rays still lights up the sky.

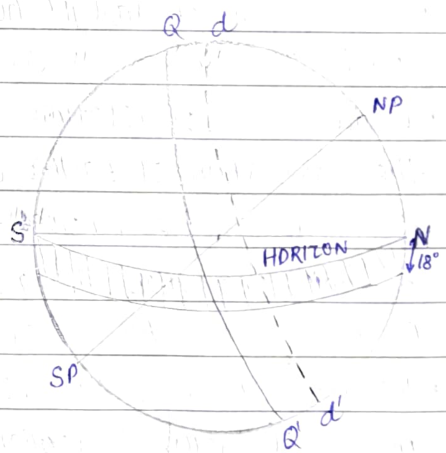
The cause of twilight are:-
- Even though the sun is below the horizon, it illuminates the upper layers of atmosphere.
- The light is reflected and scattered in various direction causing twilight.
- This scattered light illuminates the earth's surface for some time before sun rise and sunset.

* In the morning, twilight commences when the sun is 18° verticaly below the horizon and ceases at sunrise.
In the evening, twilight commences at sunset and ceases when sun is 18° verticaly below the horizon. After that, there is total darkness.

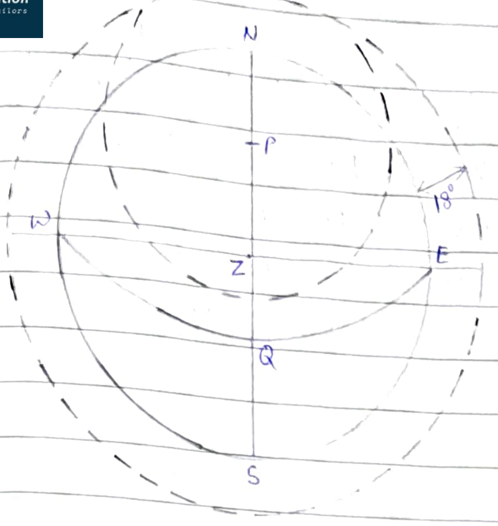
why duration of twilight varies with change of latitude

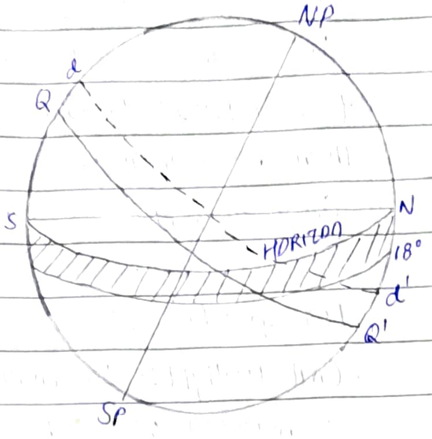
The reason why twilight last longer in higher latitude are:-




As evident in above figures, when the observer is in low latitudes, the sun rises and sets almost perpendicular to the horizon covering the 18° twilight belt over a short arc and therefore in a short period of time.

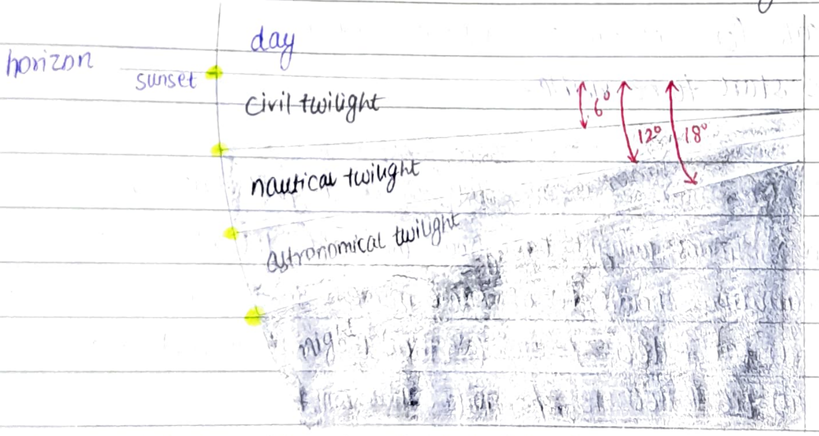



As evident in above figures, when the observer is in higher latitude, the sun rises and sets at a more oblique angle to the horizon covering the 18° twilight belt over a much larger arc and therefore over a much larger period of time.

And this is why twilight last longer in higher latitudes.

Ques 27 :- Define the terms Civil, Nautical and Astronomical twilight.




· The entire period of twilight is divided into three stages :-

- Civil - when sun is 6° below the horizon
- Nautical - when sun is 12° below the horizon.
- Astronomical - when sun is 18° below the horizon.

· In the morning, twilight commences when the sun is 18° below the horizon - first astronomical twilight commences when sun is 18° below the horizon, then nautical twilight commences when sun is 12° below the horizon, then civil twilight commences when sun is 6° below the horizon & finally twilight ceases at visible sunrise.

• In the evening, twilight commences at visible sunset – then first civil twilight continues till the sun is 6° below the horizon, then nautical twilight continues till the sun is 12° below the horizon, then astronomical twilight continues till the sun is 18° below the horizon, final twilight ceases and there is total darkness.

For specific definition :-

• Civil twilight :- The civil twilight is the period of twilight when the sun is between 0° & 6° below the horizon. The horizon is very clearly visible and sky is fairly bright. Therefore stars are not visible for stellar observation.

• Nautical Twilight : The Nautical twilight is the period of twilight when the sun is between 6° & 12° below the horizon. The horizon is clear enough for stellar observation and sky are dark enough for bright stars to be seen. Star sights are therefore best obtained during this period.

• Astronomical twilight :- The Astronomical twilight is the period of twilight when the sun is between 12° & 18° below the horizon. The horizon is too dark for celestial observation and sky are too dark for most of the stars to be visible.

(28) For Numerical Purpose, we need to just know this :-

⇒ • For continuos twilight, throughout the night, the observer's latitude & sun's declination should be of same name & limiting latitude are obtained as :-

(i) Lat + dec </= 90° (so that the sun will set), and,

(ii) Lat + dec + 18° >/= 90° (so that, will have astronomical twilight throughout the night)

(iii) Lat + dec + 12° >/= 90° ( " " , " " nautical " " " " )

(iv) Lat + dec + 6° >/= 90° ( " " , " " civil " " " " )

• The condition necessary for continuos day light : the observer's latitude & sun's declination should be of same name and the limiting latitude obtained as:

Lat + dec >/= 90° (so that sun will not set);

• The condition necessary for continuos night :- the observer's latitude & sun's declination should be of different name & limiting latitude are obtained as:

Lat + dec >/= 90° (so that sun will not rise)

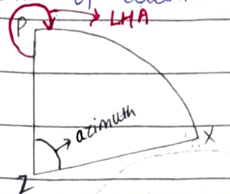
...lination is like a latitude of the celestial body
...celestial sphere, we don't have lat, long ; we have declination
...ination of a celestial body on cel. sphere = Latitude of a body on earth surface

No. 163 Date 29/04/2022

59) Explain with suitable sketch, PZX traingle.

Ans:- • The navigational traingle or PZX traingle is a spherical traingle use in navigation to determine the observer's position on celestial sphere.

• It is composed of three refrence point on celestial sphere
where, P is the celestial pole (either North or South). It is fixed point.
Z is the observer's zenith i.e. observer posⁿ on the celestial sphere.
X is the position of celestial body such as sun, moon, star & planets.

• 

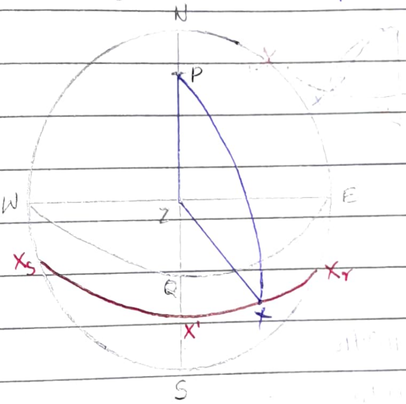

ZX = zenith distance (angular distance of celestial body from observer) = 90° − T. alt.

PX = Polar distance = 90° ± declination

∠PZX = azimuth (bearing from observer to celestial body)

~~PZ = Colatitude (connects obs. posⁿ to pole posⁿ) = 90° − latitude~~

• By measuring the angle of the celestial body in the sky, the observer can get the LHA of celestial body i.e. angle subtended at P b/w Z & X (measured westward, from observer's meridian to the meridian of celestial body)

• 


ZQ = Latitude of observer
WQE = celestial equator i.e equinoctial

• Above is a celestial sphere. Imagine watching a sphere with an observer (z) on top.

• Lets say, an observer at latitude (ZQ) observes a sun rising from east & then setting at west.

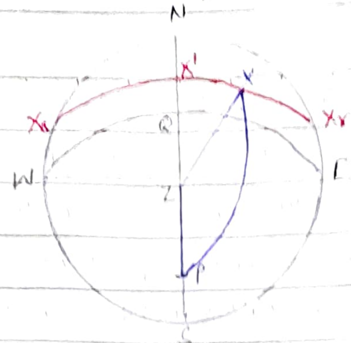
In above figure, $X_r$ = rising body ; $X_s$ = setting body
X' = body reaches at meridian passage i.e. when its meridian ~~coincides with observer's meridian.~~
X = posⁿ of celestial body

• QX' is the declination of celestial body i.e. south
ZQ is the latitude of the observer i.e. N [since the observer (z) are in north of the equinoctial (WQE)]
Lat > dec

Note:- Polar distance (PX) can be greater than 90° but ZX & PZ are always less than 90°.

e examples for understanding purpose

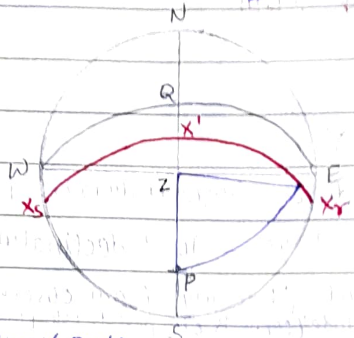
•




Here, Declination (RX') is north

& Latitude (ZQ) is south [bcz observer (z) is in south of the equinoctial (WQE)]

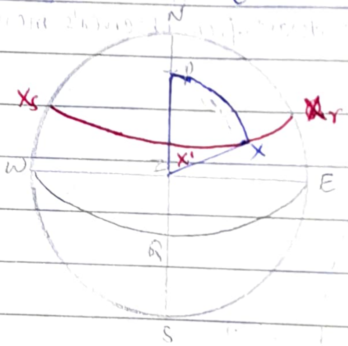
•




Here, Declination (QX') is south

& Latitude (ZQ) is south [bcz observer (z) is in south of the equinoctial (WQE)]

•




Here, Declination (QX') is north

& latitude (ZQ) is north

Dec > Lat [QX' > ZQ]

Type①:- Twilight, Rising/setting

Book P-249 ①:- If the sun's declination is 15°S, in what latitudes will there be :-

ⓐ the phenomenon of the Midnight sun (3 times)

Lat + dec ≥ 90° (so that sun will not set) ; lat & dec of same name

Lat ≥ 90° − dec

= 90° − 15°

= 75°S

So, the continuos daylight will be present in latitudes between

**75°S to 90°S**

ⓑ Twilight all night

Lat + dec ≤ 90° (so that sun will set) ; lat & dec of same name

Lat ≤ 90° − dec

= 90° − 15°

= 75°S

Lat + dec + 18° ≥ 90° (so that, will have astronomical twilight) ; lat & dec of same name

Lat ≥ 90° − dec − 18°

Lat ≥ 90° − 15° − 18°

= 57°S

So, the twilight all night will be present in latitudes between

**57°S to 75°S**

ⓒ Continuos night

Lat + dec ≥ 90° (so that sun will not rise) ; lat & dec of different name

Lat ≥ 90° − dec

= 90° − 15°

= 75°N

So, the continous night will be present in latitudes between

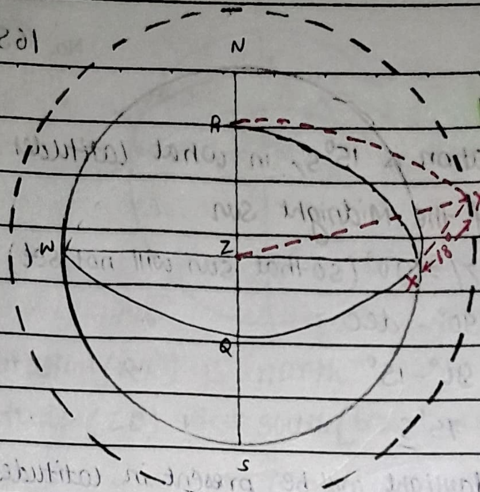
**75°N to 90°N**

Ques②:- Q-16 of book, P-247 ; 1(b) of Oct. 2021

ⓐ Calculate the duration of astronomical twilight in lat 35°N on the day of spring (vernal) equinox (3 times)

Soln:- * Astronomical twilight commences when the sun's centre is 18° below the horizon.

* also see similiar ques 2(b) on P-172



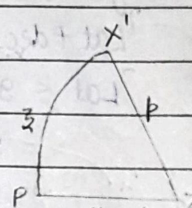

In $\Delta PZX'$: $ZX' = 90° + 18° = 108°$

$PZ = 90° - \text{Latitude}$

$\Rightarrow 90° - 35° = 55°$

Vernal equinox means declination is 0°

Using Napier's rule,




In quadrantal spherical $\Delta PZX'$,

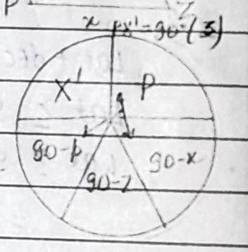
sin(midpart) = cos(opposite) x cos(opposite)

~~sin (90 − p) = cos P x cos (90 − x)~~

$\sin(90 - ZX') = \cos P \times \cos(90 - PZ)$

$\cos ZX' = \cos P \times \sin PZ$

$\cos 108° = \cos P \times \sin 55°$




$\cos P = \dfrac{\cos 108°}{\sin 55°}$

X' p Z x P
90-p 90-Z 90-x

$\cos P = -0.37724$

$P = 112° 09.7'$

Since, In $\Delta PZX$, by sine rule, angle P will be 90°

& hence ZX = 90°, PX = 90° and Z = 90°

The duration of astronomical twilight is ∠XPX'

i.e. 112° 09.7' − 90°

= 22° 09.7'

converted to time = 1 hour 28 min 39 sec

No. 167 Date 30/04/2024

**Ques ③ (a):** Q.1 of book, P-240 ; 3(b) of July 21, May 19, Mar. 19, 1(b) of July 19 (5-times)

To an observer in a certain latitude, the sun declination 12°14'N (13°17'N) bore 076°(T) (073°(T)) at theoretical sunrise, find observer's latitude. 3(b) of 3/17

**Solⁿ:-**

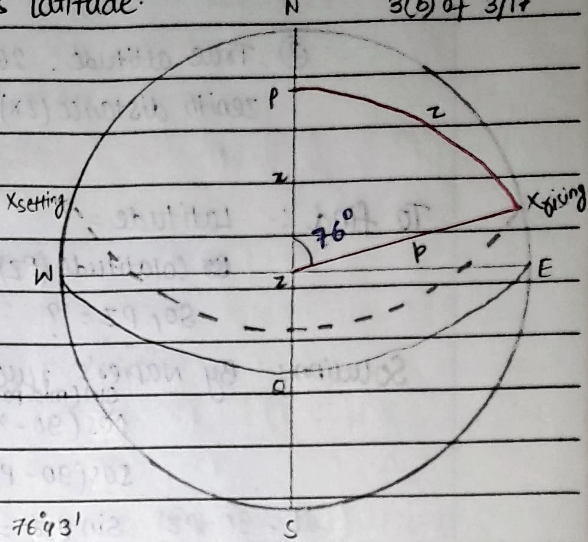
Declination : 12° 14' N

Polar distance (PX) : 90° − dec

= 90° − 12° 14'

= 77° 46'

Z = 76°




using napier's rule,

In quadrantal spherical △PZX,

~~sin (90 − Z) = cos (90 − x) · cos Z~~

sin (90 − PX) = cos (90 − PZ) · cos Z

cos PX = sin PZ · cos Z

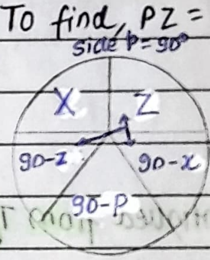
sin PZ = cos PX / cos Z

sin PZ = cos 77° 46' / cos 76° (cos 76° 43' / cos 73°)

sin PZ = 0.875875

PZ = 61° 08.9' (PZ = 51° 48.1')

To find, PZ = ?




Since, colatitude, PZ = 90° − Latitude

So, Latitude = 90° − PZ

= 90° − 61° 08.9 (8.1)

= 28° 51.1' N or S

Latitude = 38° 11.9' N or S

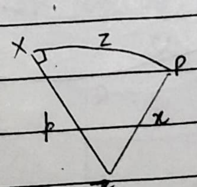
* see 3(b) on P-180, where we have to find observer's longitude.

**Type ②: Maxᵐ Azimuth**

**Ques ④:-** Q-2 of book, P-191 ; 3(b) of Oct. 2024, 9/17 (2-times)

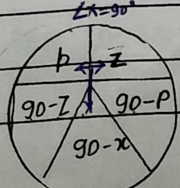
To an observer, star with declination 29° 44.6'S bore south (180°) when on the meridian. If its true altitude when at maximum azimuth was 26° 03', find the observer's latitude.

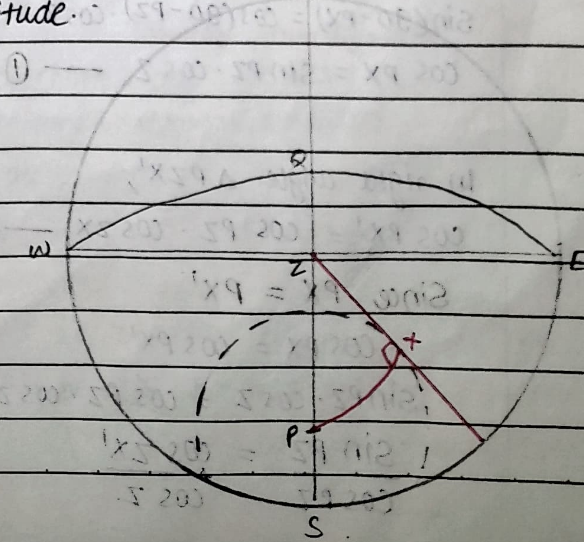
**Solⁿ:-**




| Z | ∠P | x | ∠Z | p |
|---|---|---|---|---|
| z | 90-P | 90-x | 90-Z | p |

∠X = 90°



Given :- ① Declination : 29° 44.6' S

Polar distance (PX) = 90° − dec
= 90° − 29° 44.6'
= 60° 15.4'

② True altitude : 26° 03'

zenith distance (ZX) = 90° − T. alt.
= 90° − 26° 03'
= 63° 57'

To find :- Latitude = ?

Colatitude (PZ) = 90° − Latitude ⇒ Lat = 90 − PZ

So, PZ = ?

Solution :- By Napier's rule

sin (mid part) = cos opp. × cos opp.

sin (90 − x) = cos p · cos z

sin (90 − PZ) = cos ZX · cos PX

(Lat = 90 − PZ) sin Lat = cos 63° 57' × cos 60° 15.4'

sin lat = 0.21787

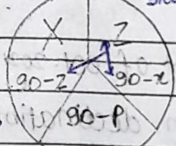
Latitude = 12° 35' (S) → bcz observer (z) is south of the equinoctial (WQE)

**Continued from Type ①**

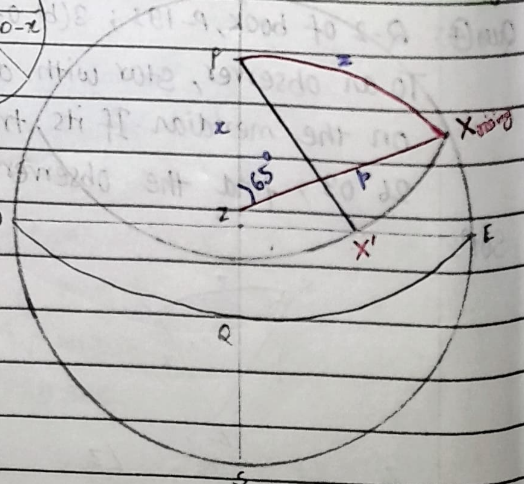
Ques ⑤ :- Q-8 of book, P-245 ; (1.b) of July 21, May 18, Sep 17 (3 times)

To an observer, star bore 065° (T) when rising, its true altitude when on prime vertical east of the meridian (bearing 90°) was 42°, find the observer's latitude & declination.

Sol$^n$ :- Using napiers rule, (in quadrantal spherical ΔPZX, side p = 90°)




sin (90 − PZ) = cos (90 − x) · cos Z




sin (90 − PX) = cos (90 − PZ) · cos Z

cos PX = sin PZ · cos Z —— ①

In right angle Δ PZX',

cos PX' = cos PZ · cos ZX' —— ⑪

Since PX = PX'

∴ cos PX = cos PX'

sin PZ · cos Z = cos PZ · cos ZX'

$$\frac{\sin PZ}{\cos PZ} = \frac{\cos ZX'}{\cos Z}$$

$\text{Tan } PZ = \dfrac{\cos ZX'}{\cos Z}$

Given :- ① ∠Z = 65°

② T. alt = 42°

Zenith distance, ZX' = 90° − T. alt.

= 90° − 42°

= 48°

To find : Latitude = ?

Colatitude (PZ) = 90° − Latitude

So, PZ = ?

Solution: $\text{Tan } PZ = \dfrac{\cos ZX'}{\cos Z}$

$\text{Tan } PZ = \dfrac{\cos 48°}{\cos 65°}$

Tan PZ = 1.5832979

PZ = 57° 43.4'

* Declination can be find by ~~putting the values in equation①~~

Since, PZ = 90° − Lat

So, Latitude = 90° − PZ

= 90° − 57° 43.4'

= 32° 16.6' (N) → bcz observer (Z) is north of equinoctial (WQE)

Ques⑥:- Ques① of book, P-248 ; 2(b) of July 2024, May 2019 (2 times)

In what latitude would the longest day be 5 hours more than the shortest day?

Soln:- Note:- Normally, at vernal equinox we have 12 hours of day & 12 hours of night. So, a shortest duration of day is 12 hours. So if I have a longest day is 5 hours more than shortest day & for example if shortest day is 12 hrs (x), Then longest day will be 12+5 = 17 (x+5)

Longest day + shortest day = 24 hrs — Just for understanding

So, lets assume, the duration of shortest day = x hrs

So, " " " longest day = x + 5 hrs

x + (x + 5) = 24

2x + 5 = 24

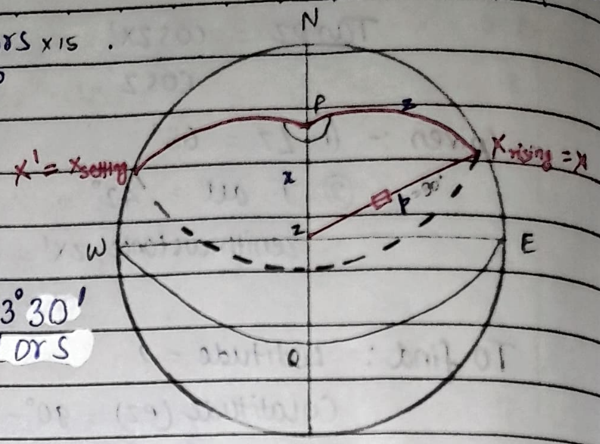
2x = 19

x = 9.5 hrs i.e. shortest day.

So, the longest day will be 9.5 + 5 = 14.5 hrs (only if longest day be 5 hours more than shortest day)

No. 170 Date 30/04/2024

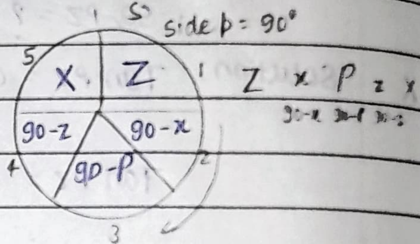
- The angle at P b/w X & X' = 14.5 hrs x 15
  = 217.5°



Hence, ∠XPZ (∠P) = 217.5/2
= 108° 45'

- On the longest day, sun will have maximum declination i.e. of 23° 30' N or S

- Using Napier's rule,
  In quadrantal spherical ΔPZX,



sin(midpart) = Tan(adjacent) x Tan(adjacent)
sin(90-P) = Tan(90-z) x Tan(90-x)
cos P = Tan(90-PX) x Tan(90-PZ)
cos P = Tan dec x Tan lat

$$\text{Tan lat} = \frac{\cos P}{\text{Tan dec}}$$

$$\text{Tan lat} = \frac{\cos 108°45'}{\text{Tan } 23°30'}$$

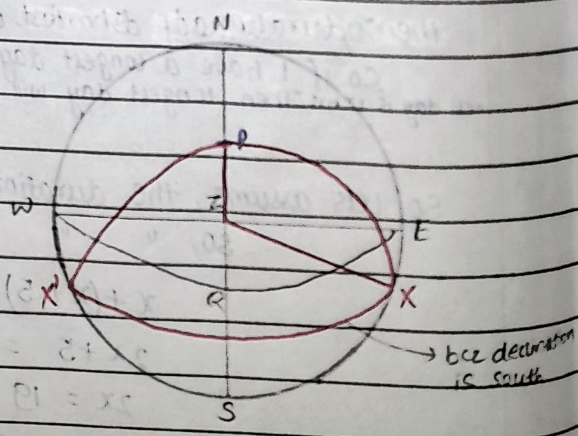
Tan lat = −0.73926

Latitude = 36° 28.4' N or S

Ques 7 (a) Ques 6 of book P-243 (Not asked yet)

Required the latitude in which the period of darkness will be twice the period of daylight, when the Sun's declination is 22° 40'S

Sol^n: Darkness : daylight
2 : 1
i.e. 16 hours : 8 hours



- The angle at P b/w X & X' = 8 hrs x 15
  = 120°

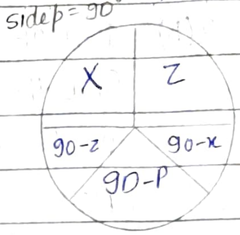
Hence ∠XPZ (∠P) = 120/2
= 60°

- Declination : 22° 40'S

No. 171 Date 30/04/2021

Using Napier's rule,

In quadrantal spherical $\Delta PZX$,




$\sin(\text{midpart}) = \tan(\text{adjacent}) \times \tan(\text{adjacent})$

$\sin(90-P) = \tan(90-z) \times \tan(90-x)$

$\cos P = \tan(90-PX) \times \tan(90-PZ)$

$\cos P = \tan dec \times \tan lat$

$\tan lat = \dfrac{\cos P}{\tan dec}$

$\tan lat = \dfrac{\cos 60°}{\tan 22°40'}$

$\tan lat = 1.19724$

Latitude = 50° 07.8' (N) → bcz declination is S i.e. winter soltice in which shorter day is in Northern Hemisphere

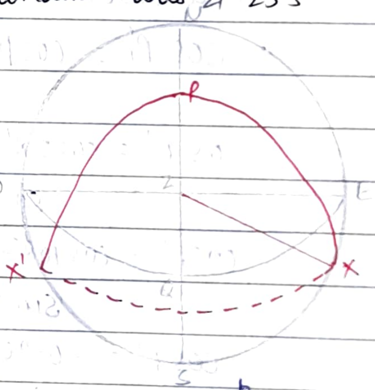
Q.2(a) of Nov 2021, May 2017 (2 times)

#(b) Find the ratio of the period of darkness to the period of daylight in latitude 52°15'N when the sun's declination was 21°25'S

Sol^n:- Given :- Latitude : 52°15'N

Declination : 21° 25'S

To find :- $\angle P$ = ?




Solution :

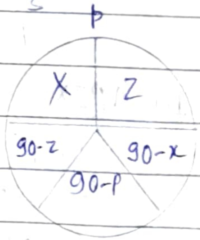
Using Napier's rule,

In quadrantal spherical $\Delta PZX$,

$\sin(\text{midpart}) = \tan(\text{adjacent}) \times \tan(\text{adjacent})$

$\sin(90-P) = \tan(90-z) \times \tan(90-x)$

$\sin(90-P) = \tan(90-PX) \times \tan(90-PZ)$




$\cos P = \tan dec \times \tan lat$

$\cos P = \tan 21°25' \times \tan 52°15'$

$\cos P = 0.50657$

$P = 59° 33.8'$

So, Angle at P b/w X and X' = 59° 33.8' × 2

= 119° 7.6'

= 7h 56m 30s → i.e. period of daylight

= 7.9 hrs

So period of darkness = 24 hrs − 7h 56m 30s

= 16h 3m 30s = 16.1 hrs

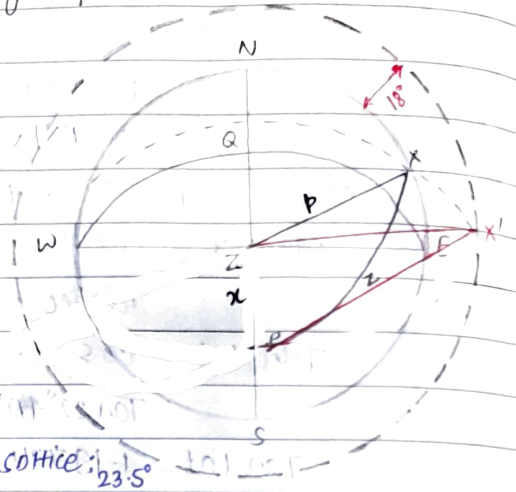
Darkness : daylight

16.1 : 7.9

2.037 : 1

2(b) 1(b) of March 2024 & Jan 2024 (2 times)

Find the duration of astronomical twilight for an observer in Lat 20°00'S on the day of summer soltice.




Sol$^n$:- * Astronomical twilight commences when sun's centre is 18° below the horizon

In $\Delta PZX'$, • $ZX' = 90° + 18° = 108°$

• Latitude : 20°00'S

$PZ = 90° - lat$

$= 90° - 20°$

$= 70°$

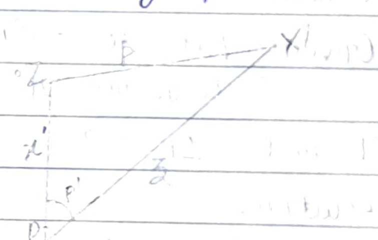
• Declination of sun at summer soltice: 23.5°

$PX' = 90° + dec$

$\Rightarrow 90° + 23.5° = 113.5°$

In $\Delta PZX'$,

Since there is no side or angle that is 90°, that is why napier's rule cannot apply here



$$\cos P' = \frac{\cos p - \cos x' \times \cos z}{\sin x' \times \sin z}$$

$$\cos P' = \frac{\cos ZX' - \cos PZ \times \cos PX'}{\sin PZ \times \sin PX'}$$

$$\cos P' = \frac{\cos 108° - (\cos 70° \times \cos 113.5°)}{\sin 70° \times \sin 113.5°}$$

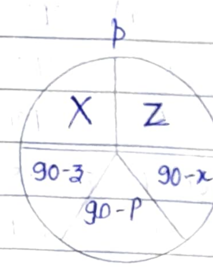
$\cos P' = -0.20033$

$P' = 101° 33.4'$

In $\Delta PZX$,

Using napier's rule,

In quadrantal spherical $\Delta PZX$,




sin(midpart) = Tan(adjacent) x Tan(adjacent)

$\sin(90 - P) = \tan(90 - z) \times \tan(90 - x)$

$\cos P = \tan(90 - PX) \times \tan(90 - PZ)$

$\cos P = \tan dec \times \tan lat$

$\cos P = \tan 23.5° \times \tan 20°$

$\cos P = 0.15826$

$P = 80° 53.6'$

The duration of astronomical twilight is

$\angle ZPX' - \angle ZPX$

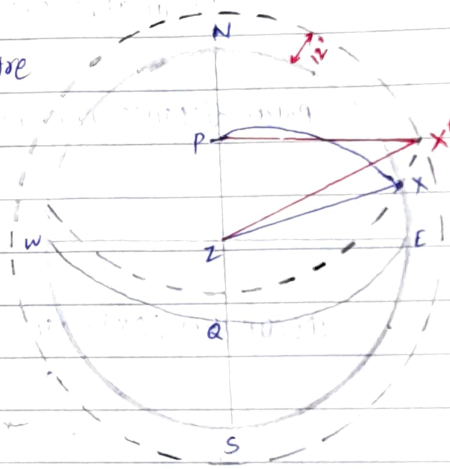
$= 101°33.4' - 80°53.6'$

$= 20°39.8' \div 15$

= 1hr 22m 39sec

Ques(8):- Calculate the period of Nautical Twilight for an observer in latitude 30°N and declination of sun 20°N

Soln:- Nautical Twilight commences when sun's centre is 12° below the horizon.




In $\Delta PZX'$, $ZX' = 90° + 12° = 102°$

- Latitude : 30°N
  $PZ = 90° - Lat$
  $\Rightarrow 90° - 30° = 60°$
- Declination : 20°N
  $PX = 90 - dec$
  $PX' = 90 + dec$
  $PX' = 90 + 20$
  $= 110°$

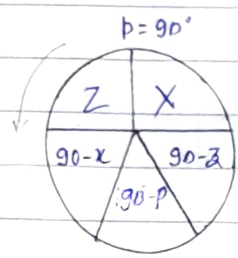
In $\Delta PZX'$, since there is no side or angle that is 90°, that is why Napiers rule cannot apply here

$$\cos P' = \frac{\cos p - \cos x \times \cos z}{\sin x \times \sin z}$$

$$\cos P' = \frac{\cos ZX' - \cos PZ \times \cos PX'}{\sin PZ \times \sin PX'}$$

$$\cos P' = \frac{\cos 102° - (\cos 60° \times \cos 110°)}{\sin 60° \times \sin 110°}$$

$$P' = 92°35.9'$$




In $\Delta PZX$, using napier's rule, $ZX = p = 90°$

$\sin(90-P) = \tan(90-z) \times \tan(90-x)$

$\sin(90-P) = \tan(90-PX) \times \tan(90-PZ)$

$\cos P = \tan lat \cdot \tan dec$

$\cos P = \tan 30° \times \tan 20°$

$P = 77°52.2'$

The duration of Nautical Twilight $= \angle ZPX' - \angle ZPX$

$= 92°35.9' - 77°52.2'$

$= 14°43.7' \div 15$

= 0hr 58m 54sec.

180° − (SX + NX') ↑

**Type ③ :- Circumpolar bodies**

**Ques ⑨ :-** Q.1 of Book P-183; 3(b) of Dec 2020, Nov 18 (4 times), 7/17, Jan16

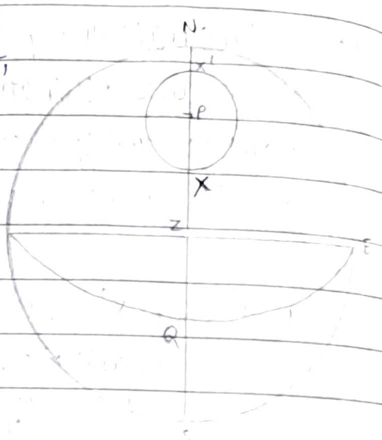
A star when on meridian above the pole, bore North with a true altitude of 70° 04' and when on the meridian below the pole, bore North with true altitude 22° 05'. Find the observer's latitude & the star's declination.

**Soln:-**

Upper meridian altitude, NX = 70° 04'

Lower meridian altitude, NX' = 22° 05'

Declination circle (XX') = NX − NX' = 47° 59'




Polar distance (PX or PX') = XX'/2

= 47° 59'/2

= 23° 59.5'

Declination (QX) = 90° − Polar distance (PX or PX')

= 90° − 23° 59.5'

= 66° 00.5' N

Observer's latitude (QZ) = Altitude of pole (NP) = NX' + PX'

= 22° 05' + 23° 59.5'

= 46° 04.5' N

* Just for knowledge

**Note:-** If the altitudes of a circumpolar body at upper and lower meridian passage are observed

- then the observer's latitude and
- body's declination can be calculated

Steps:-

ⓐ Draw a rational horizon, place the body at upper transit (X) & lower transit (X')

ⓑ Place elevated pole midway beetween the 2 positions and draw declination circle, with the pole as the center and circle passing through X and X'

ⓒ Draw equinoctial, WQE, where PQ is 90°.

* ⓓ Diameter of declination circle (XX')

= Upper altitude (NX) − lower altitude (NX')
(when upper & lower mer. pass, brg is same)

= 180° − (Upper altitude (SX) + lower altitude (NX'))
(when upper & lower mer. pass, brg is opposite)

Smart Mariner Edition
for smart sailors

No. 175 Date 01/05/2022

(e) The diameter divided by 2 gives the polar distance PX and PX'

$(PX \text{ or } PX') = XX'/2$

(f) Polar distance (PX) = altitude of pole (NP) - lower meridian altitude (NX')

* Polar distance (PX) = 90° - declination.

(g) Polar distance (PX) + lower meridian altitude (NX') = longitude of observer, named same as elevated pole

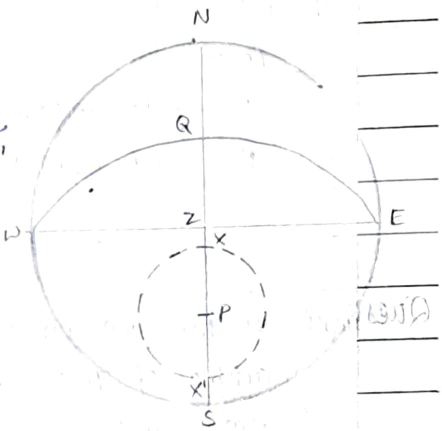
Ques(10):- Q-2 of BOOK P-184; 3(a) of oct 2020

During the same night, a star bore south with true altitude 28°34' and again with a true altitude 76°46'. Calculate the star's declination and the latitude of the observer.

Soln:- (Upper transit) Upper meridian altitude, SX = 76°46'

(lower transit) lower meridian altitude, SX' = 28°34'

Declination circle (XX') = SX - SX' = 48°12'




Polar distance (PX or PX') = XX'/2

= 48° 12'/2

= 24° 6'

Declination (QX) = 90° - PX

= 90° - 24° 6'

= 65° 54' S

Latitude (QZ) = Altitude of Pole (SP) = SX' + PX'

= 28°34' + 24° 6'

= 52° 40' S

Ques(11) A star with declination 52° 12' S had a true altitude of 24° 15' at lower transit. Find the sextant altitude of the same star at upper transit I.E 1.5' off the arc. HE - 10 m.

Soln:- Declination = 90° - PX

Polar distance (PX) = 90° - dec

= 90° - 52° 12'

= 37° 48'

PX = XX'/2

Declination circle, XX' = 2 PX

= 2 X 37° 48'

= 75° 36'

XX' = SX − SX'

i.e. Declination circle (XX') = upper transit (SX) − lower transit (SX')

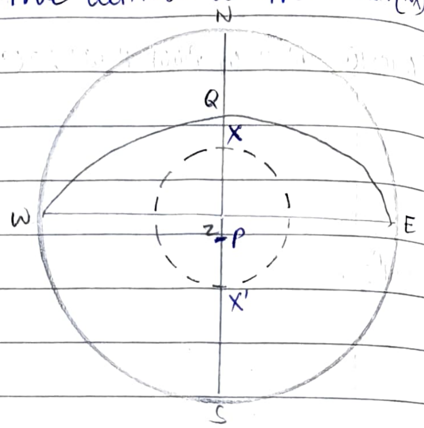
75° 36' = SX − 24° 15'

Upper transit (SX) = 99° 51'

= 180° − 99° 51'

NX = 80° 09' i.e. true altitude at upper transit (NX)

True altitude : 80° 09'
Total correction : + 0.2'
Apparent altitude : 80° 9.2'
Dip (HE − 10 m) : + 5.6'
Observed altitude : 80° 14.8'
IE (off) : − 1.5'
Sextant altitude : 80° 13.3'




Ques (12). Q3 of BOOK P-186; 2(b) of 3/17, 9/16, 7/16, (3 times)

Find two latitudes in which a star having a declination of 68° 46'N (68° 51' S) will bear North (South) with true altitude of 16° 12' (11° 50').

Soln:- For a star to bear the same (i.e. North) and have the same altitude when on meridian

Latitude will be above the pole in one case & below the pole in one case.

(i) NX = 16° 12'

Declination, QX = 68° 46'

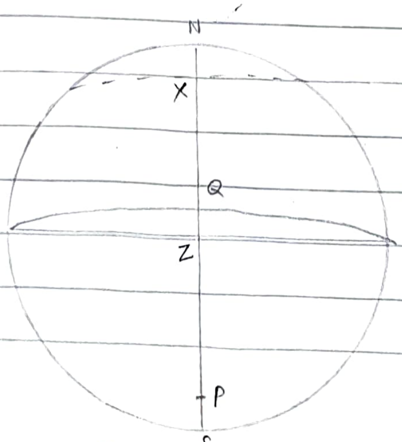
NQ = NX + QX

= 16° 12' + 68° 46' = 84° 56'

Latitude (QZ) = 90° − NQ

= 90° − 84° 56'

= 05° 02' S




Above the pole

No. 177 Date 01/05/2022

(ii) $NX' = 16°12'$

Declination (QX) = $68°46'$

Declination (QX) = $90°$ − Polar distance (PX')

$PX' = 90 - QX$

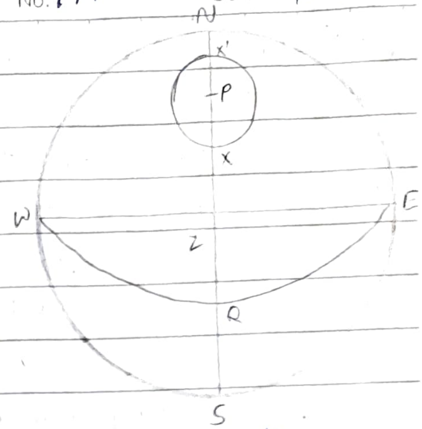
$PX' = 90 - 68°46'$

$PX' = 21°14'$

Latitude (NP) = $NX' + PX'$

$= 16°12' + 21°14'$

$= 37°26'N$




Below the pole

**

Lat: 45°S, Dec: 50°S ↑ Lat: 50°N, Dec: 56°N ↑

Ques (13) :- 1(b) of Sep 18, 11/16, 9/16; Similiar 3(b) of Aug 21, Nov 20 (5 times)

If the Latitude was 64°27'S (61°35'S) and declination of a star was 39°47'S (34°43'S). Find out if the body is circumpolar. If so calculate the upper and lower meridian altitude.

Sol$^n$:- ✱ Condition required for a body to be circumpolar

- Lat + dec ≥ 90° (For the body not to set)
- Lat & dec should be of same name.

- So here, Lat + dec ⇒ $64°27' + 39°47' = 104°14'$ i.e. ≥ 90°
  & Lat & dec are both of same name i.e. south,
  therefore body is circumpolar.

- Polar distance (PX or PX') = $90°$ − declination
  ⇒ $90° - 39°47' = 50°13'$

  Declination circle (XX') = 2·PX
  ⇒ $2 \times 50°13' = 100°26'$

- Altitude of pole (NP) = Observer's Latitude (QZ) = $SX' + PX'$

  $64°27' = SX' + 50°13'$

  Lower transit (SX') = $64°27' - 50°13'$
  $= 14°14'$

  Declination circle (XX') = $180° - (NX + SX')$

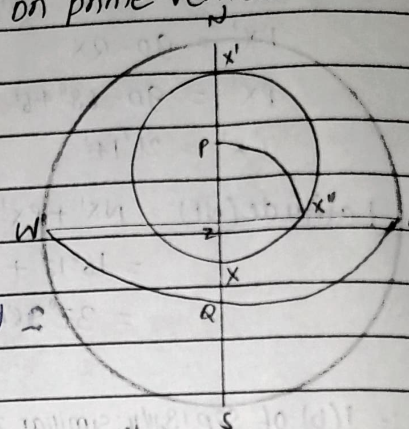
  Upper transit (NX) = $180° - (XX' + SX')$

  $= 180° - (100°26' + 14°14')$

  $= 180° - 114°40'$

  $= 65°20'$

**Ques(14):-** To an observer, the true altitude of a star when on the meridian at upper and lower transit were 80° 09'S and 11° 45'N respectively. Calculate the true altitude when on prime vertical.




**Ans:-** Upper transit (SX) = 80° 09' S

lower transit (NX') = 11° 45' N

SX + NX' = 91° 54'

- Since, upper & lower are of diff. name,

Declination circle (XX') = 180° − (SX + NX')

= 180° − 91° 54'

= 88° 06'

Polar distance (PX or PX') = XX'/2

= 88° 06'/2

= 44° 03'

- Declination (QX) = 90° − Polar distance (PX or PX')

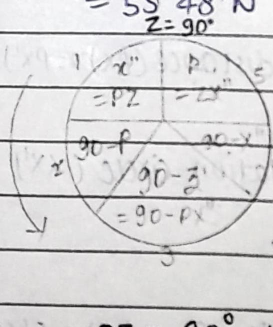
= 90° − 44° 03'

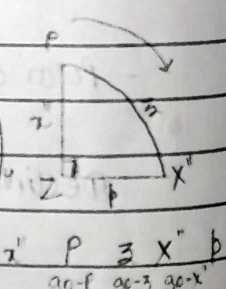
= 45° 57' N

- Latitude (QZ) = Altitude of pole (NP) = NX' + PX'

= 11° 45' + 44° 03'

= 55° 48' N




- In right angle ΔPZX'', Z = 90°

By Napier rule,

sin(midpart) = cos(opposite) x cos(opposite)

sin (90 − PX'') = cos PZ x cos ZX''

cos PX'' = cos PZ x cos ZX''

cos ZX'' = cos PX'' / cos PZ

cos ZX'' = cos 44° 03' / cos 34° 12'

PZ = 90° − Latitude

= 90° − 55° 48'

= 34° 12'

cos ZX'' = 0.869

ZX'' = 29° 39.4'

- ZX'' = 90° − T alt

True altitude = 90° − ZX''

= 90° − 29° 39.4'

= 60° 20.6'

upper = x
lower = x' — bearing north — x' will be upside

3(b) of July 18 (1 times) No. 179 Date 01/05/2022

Ques(5):- In latitude 65°N, an observer's obtain a lower meridian altitude of a celestial body as 20° bearing North. Calculate the altitude and bearing of the same celestial body at upper meridian passage.

Sol^n:- Given:-

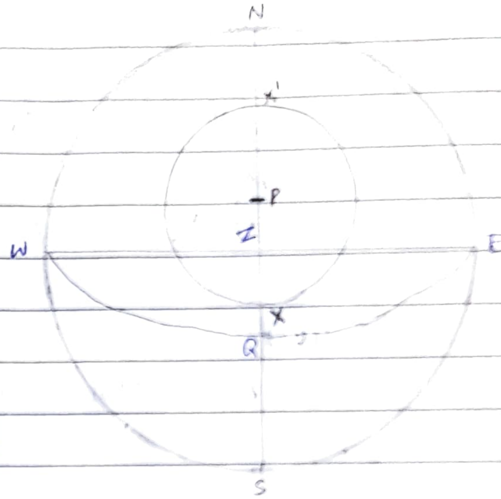
Latitude (QZ) = Altitude of pole (NP) = 65°

Lower meridian altitude (NX') = 20°

PX' = NP − NX' ⇒ 65° − 20° = 45°

Declination (XX') = 2·PX ⇒ 2×45° = 90°

To find :- SX = ?




Solution:-

Declination (XX') = 180° − (SX + NX')

XX' = 180° − SX − NX'

SX = 180° − XX' − NX'

SX = 180° − 90° − 20°

SX = 70°

Hence, altitude of body at upper meridian passage is 70° bearing south.

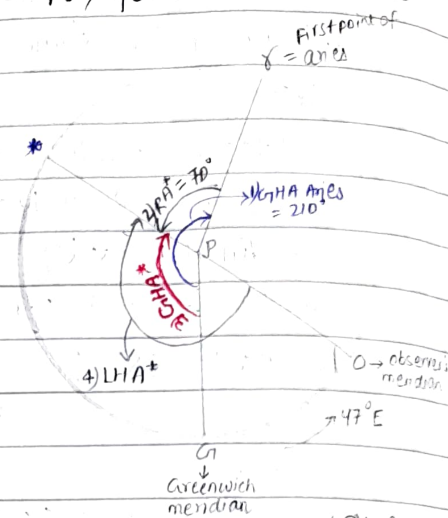
## Type ④: LHA, GHA

Ques ⑯:- Q 1 of Book P-69 ; 1(b) of Sep 21 & similiar 2(b) of Nov 21 (2 times)

Calculate the LHA of a star whose RA is 70°, for an observer in longitude 47°E, when GHA♈ is 210°.

Soln:-

GHA ♈ = 210°
RA* = 70°
GHA* = 140°
Long (E) = 47°
LHA* = 187°




Answer:-

LHA of the star is 187°

* GHA measured westward (↻) from greenwich to body
* RA contained b/w celestial meridian of First point of aries & body (star), measured eastward
* LHA contained b/w celestial mer. of observer to body, measured westward

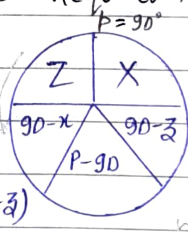
### Continued from Type 1

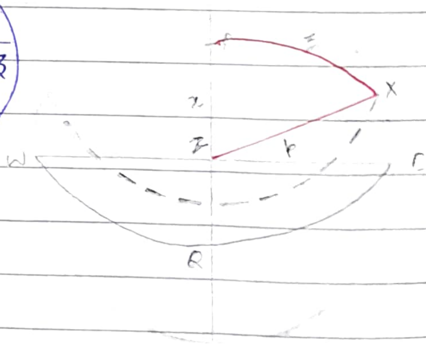
3(b) Q2 of book P-241, similiar ques. - 2(b) of July 19 (1 times)

In latitude 37°38'N, at theoretical sunrise, the sun had a declination of 22°01'N and GHA 112°13'. Requires the observer's longitude.

Soln:- In $\Delta PZX$, $ZX = p = 90°$

- Using Napier's rule,



$\sin(\text{midpart}) = \tan(\text{adj.}) \times \tan(\text{adj.})$

$\sin(P - 90°) = \tan(90 - x) \times \tan(90 - 3)$

$-\cos P = \tan(90 - PZ) \times \tan(90 - PX)$

$\cos(180° - P) = \tan \text{Lat} \times \tan \text{dec}$

$\cos(180° - P) = \tan 37°38' \times \tan 22°01'$

$\cos(180° - P) = 0.3117776$

$180° - P = 71°50'$

Z x P 3 X
90-x P-90 90-3

- LP i.e. EHA ⇒ 180° − 71°50' = 108° 10'
- LHA Sun = 360° − P ⇒ 360° − 108° 10' = 251° 50'

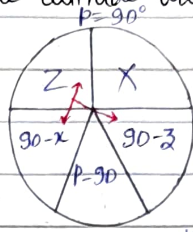
- LHA Sun : 251° 50'
GHA Sun : 112° 13'
Longitude : 139° 37' E

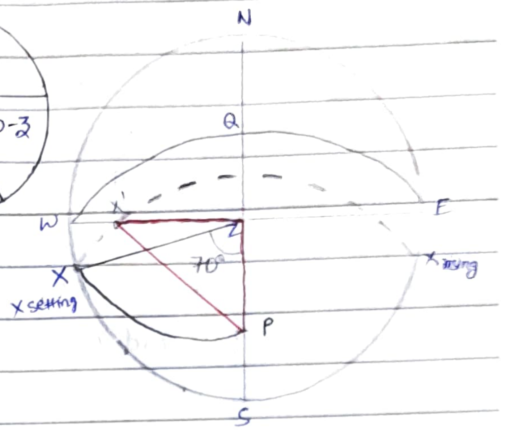
LHA − GHA = obs. long (E)
GHA − LHA = obs. long (W)

No. 181 Date 02/05/2022

Ques(17) 3 of Sep 19, (2 times) also refer to Q-3 of book P-241

For a stationary observer, amplitude of the setting sun was W 20° S. When it was on prime vertical (bearing 90°), its true altitude was 9°. Find the latitude of observer & declination of sun.




Soln:- Using napier's rule,

In $\Delta PZX$, $ZX = p = 90°$,

sin(midpart) = cos(opposite) × cos(opposite)

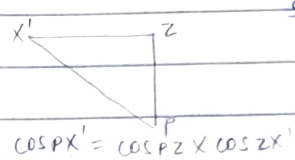
$\sin(90-3) = \cos(90-x) \times \cos Z$

$\sin(90-PX) = \cos(90-PZ) \times \cos Z$

$\cos PX = \sin PZ \times \cos Z$ — ①

- In right angle $\Delta PZX'$,

$\cos PX' = \cos PZ \cdot \cos ZX'$ — ⑪




- Since $PX = PX'$

$\cos PX = \cos PX'$

$\sin PZ \cdot \cos Z = \cos PZ \cdot \cos ZX'$

$\frac{\sin PZ}{\cos PZ} = \frac{\cos ZX'}{\cos Z}$

$\tan PZ = \frac{\cos ZX'}{\cos Z}$

- Given:- ① $\angle Z = 90° - 20° = 70°$

② T. alt = 9°

zenith distance (ZX') = 90° − T. alt
= 90 − 9°
= 81°

- To find: Latitude = ? for that PZ = ?

Declination = ?

- Solution: $\tan PZ = \frac{\cos ZX'}{\cos Z}$

$\tan PZ = \cos 81° / \cos 70°$

$\tan PZ = 0.457384$

$PZ = 24° 34.7'$

Since, colatitude (PZ) = 90° − Lat

Lat = 90° − PZ

= 90° − 24° 34.7'

= 65° 25.3' S

$\sin \text{Amplitude} = \sin \text{declination} \times \sec \text{latitude}$

$\sin \text{Amp} = \dfrac{\sin \text{dec}}{\cos \text{lat}}$ (we can also find declination by putting values in equation ①)

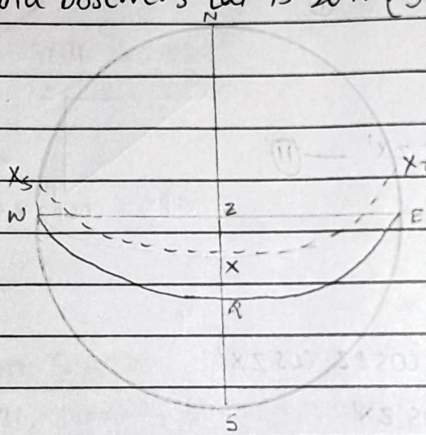
$\sin \text{dec} = \sin \text{Amp} \times \cos \text{lat}$

$\sin \text{dec} = \sin 20^\circ \times \cos 65^\circ 25.3'$

$\sin \text{dec} = 0.14226$

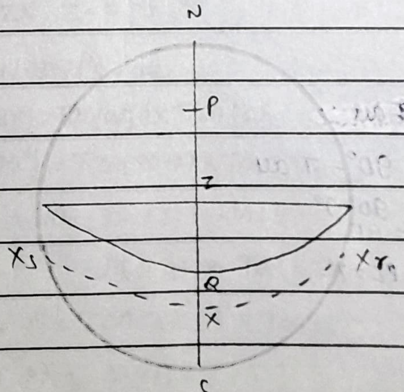
$\text{Dec} = 8^\circ 10.7' S$

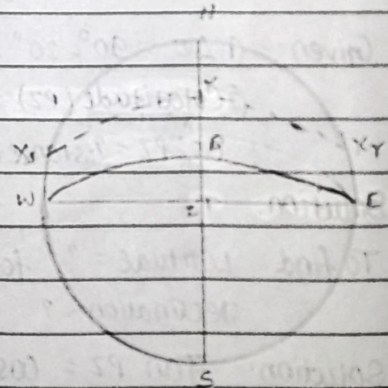
Ques ⑱:- Sketch and show Rational Horizon diagram for a rising body whose dec is 10°N and observer's lat is 20°N (3 times)




Latitude = QZ = 20°N
Declination = QX = 10°N

⇒




Lat (QZ) = 10°N
Dec (QX) = 20°S

Lat (QZ) = 10°S
Dec (QX) = 20°N